Name: Tom

Worksheet #21

Math 221

Instructions. Put the your first and last name at the top of your paper. Everyone is to do their own worksheet but only one from each group is graded with the score shared. Be sure to **explain your reasoning**.

1. (4 points each) Suppose that f is integrable on the interval $[2, 12]$. Given that $\int_2^{12} f(x)\,dx = 25$, $\int_2^8 f(x)\,dx = 10$ and $\int_4^{12} f(x)\,dx = 22$, evaluate the following definite integrals.

   (a) $\int_8^2 f(x)\,dx$

   $= -\int_2^8 f(x)\,dx$

   $= \boxed{-10}$

   (b) $\int_2^4 f(x)\,dx$

   $\int_2^4 f(x)\,dx + \int_4^{12} f(x)\,dx = \int_2^{12} f(x)\,dx$

   $\Rightarrow \int_2^4 f(x)\,dx = 25 - 22$

   $= \boxed{3}$

   (c) $\int_4^8 2 - f(x)\,dx = \int_4^8 2\,dx - \int_4^8 f(x)\,dx$

   $= 2 \cdot 4 - \left(\int_2^8 f(x)\,dx - \int_2^4 f(x)\,dx\right)$

   $= 8 - 10 + 3$

   $= \boxed{1}$

Use the Fundamental Theorem of Calculus to compute the following:

2. A particle moving along the axis has velocity $v(t) = t^2 - t$ units/s at time t in seconds. How far does the particle travel in the first second? How far does the particle travel in 2 seconds?

First second: $\int_0^1 (t^2 - t)\,dt = \frac{1}{3}t^3 - \frac{1}{2}t^2\Big|_0^1 = \frac{1}{3} - \frac{1}{2} = -\frac{1}{6}$ units

But we want distance (how far), which must be positive,

$\left|-\frac{1}{6}\right| = \frac{1}{6}$ units

2 seconds: Be careful! To get total distance we need each piece to be positive

$\left|\int_0^1 (t^2 - t)\,dt\right| + \left|\int_1^2 (t^2 - t)\,dt\right| = \frac{1}{6} + \left(\frac{1}{3}t^3 - \frac{1}{2}t^2\right)\Big|_1^2 = \frac{1}{6} + \left(\frac{8}{3} - \frac{4}{2} - \frac{1}{3} + \frac{1}{2}\right)$

$= \frac{1}{6} + \frac{7}{3} - \frac{3}{2}$

$= \boxed{1}$

Use the Fundamental Theorem of Calculus to compute the following:

3. $\frac{d}{dx}\left(\int_{-\sqrt{x}}^{20} \cos t\, dt\right) = \frac{d}{dx}\left(\sin(20) - \sin(-\sqrt{x})\right) = 0 - \cos(-\sqrt{x}) \cdot \left(-\frac{1}{2}x^{-\frac{1}{2}}\right)$

Just a number

So derivative is zero!

$= \frac{\cos(-\sqrt{x})}{2\sqrt{x}}$

$f(t) = \cos(t)$

$F(t) = \sin(t)$

$\int_a^b f(t)\,dt = F(b) - F(a)$

4. $\int_0^{\frac{\pi}{2}} \sin x\, dx = -\cos(x)\Big|_0^{\frac{\pi}{2}} = -\underbrace{\cos\left(\frac{\pi}{2}\right)}_{0} - \left(-\underbrace{\cos(0)}_{1}\right) = 1$