Breaking Bad

Name: ____________________

Worksheet #20

Math 221

Instructions. Put the your first and last name at the top of your paper. Everyone is to do their own worksheet but only one from each group is graded with the score shared. Be sure to **explain your reasoning.**

1. Determine the general anti-derivative of $5x^4 - \sqrt{x} - cos(x)$.

$x^5 - \frac{2}{3}x^{3/2} - \sin(x) + C$

Check by taking derivative!

2. Express the limit $\lim_{n\to\infty} \sum_{i=1}^{n} \Delta x\underbrace{(3 - 4x_i)}_{f(x_i)}$ on the interval $[1, 4]$ as a definite integral.

a b $\int_a^b f(x)\,dx$

$\int_1^4 (3-4x)\,dx$

3. Evaluate the limit from the previous question. Please be very careful with notation on this problem.

Need to know $\Delta x = \frac{b-a}{n} = \frac{4-1}{n} = \boxed{\frac{3}{n}}$

$x_i = a + i\Delta x = \boxed{1 + \frac{3i}{n}}$ (right endpoint)

$\lim_{n\to\infty} \sum_{i=1}^{n} \Delta x(3-4x_i) = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\left(3-4\left(1+\frac{3i}{n}\right)\right)$

$= \lim_{n\to\infty} \left(\frac{3}{n} \sum_{i=1}^{n} \left(-1 - \frac{12i}{n}\right)\right)$ Now split the summation

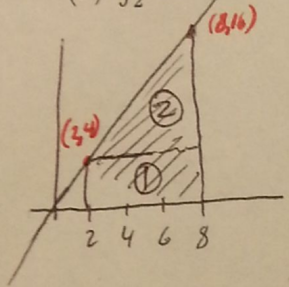
$= \lim_{n\to\infty} \left(\frac{3}{n} \underbrace{\sum_{i=1}^{n}(-1)}_{=-n} - \frac{3}{n}\sum_{i=1}^{n}\left(\frac{12i}{n}\right)\right)$ Factor out $\frac{12}{n}$

$= \lim_{n\to\infty} \left(\frac{3}{n}(-n) - \frac{36}{n^2}\overbrace{\sum_{i=1}^{n} i}^{=\frac{n(n+1)}{2}}\right) = \lim_{n\to\infty} \left(-3 - \frac{36}{n^2}\cdot\frac{n(n+1)}{2}\right) = -3 - 18$ now take the limit.

$= \boxed{-21}$

4. For each of the following integrals,
   1) Make a sketch of the area being represented,
   2) Compute the area using geometry if convenient,
   3) Compute a Riemannian approximation using 4 pieces and the right hand end points,
   4) Compute the integral as limit of the Riemannian sums as n goes to ∞.

(a) $\int_2^8 2x\,dx$



$A = 1 + 2 = 6 \cdot 4 + \frac{1}{2} \cdot 6 \cdot 12 = 24 + 36 = \boxed{60}$

$R_4 = \frac{8-2}{4}\left(2\left(\underbrace{2+\frac{6}{4}}_{x_1}\right) + 2\left(\underbrace{2+2\cdot\frac{6}{4}}_{x_2}\right) + 2\left(\underbrace{2+3\cdot\frac{6}{4}}_{x_3}\right) + 2\left(\underbrace{2+4\cdot\frac{6}{4}}_{x_4}\right)\right)$

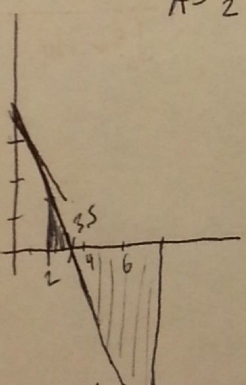
$= \boxed{\underbrace{\frac{6}{4}}_{\Delta x}\left(2\cdot(3.5) + 2(5) + 2(6.5) + 2(8)\right)}$

$\lim_{n\to\infty} \sum_{i=1}^{n} 2\left(2+i\cdot\frac{6}{n}\right)\cdot\frac{6}{n} = \lim_{n\to\infty}\left(\frac{24}{n}\sum_{i=1}^{n} 1 + \frac{72}{n^2}\sum_{i=1}^{n} i\right) = \lim_{n\to\infty} 24 + \frac{72}{n^2}\cdot\frac{n(n+1)}{2} = 24+36$

$= \boxed{60}$

(b) $\int_2^8 7 - 2x\,dx$

$A = \frac{1}{2}(1.5)(3) + \frac{1}{2}(4.5)(-9)$



$R_4 = \frac{6}{4}\left((7-2(3.5)) + (7-2(5)) + (7-2(6.5)) + (7-2(8))\right)$

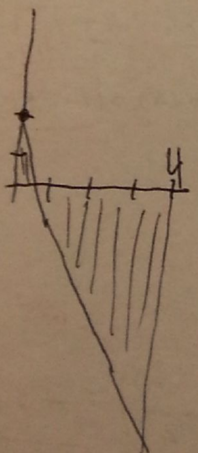
same $x_1\ x_2\ x_3\ x_4$ as (a)

$\lim_{n\to\infty} \sum_{i=1}^{n}\left(7-2\left(2+i\frac{6}{n}\right)\right)\frac{6}{n} = \lim_{n\to\infty}\left(\frac{18}{n}\sum_{i=1}^{n} 1 - \frac{72}{n^2}\sum_{i=1}^{n} i\right) = \lim_{n\to\infty}\left(18 - \frac{72}{n^2}\frac{n(n+1)}{2}\right)$

$= 18 - 36 = \boxed{-18}$

(c) $\int_0^4 2 - 3x\,dx$

$\Delta x = \frac{4}{n}$



$A = \frac{1}{2}\left(\frac{2}{3}\right)(2) + \frac{1}{2}\left(\frac{10}{3}\right)(-10)$

$R_4 = \frac{4-0}{4}\left(2-3(1) + 2-3(2) + 2-3(3) + 2-3(4)\right)$

$= \left(2-3+2-6+2-9+2-12\right)$

$\lim_{n\to\infty} \sum_{i=1}^{n}\left(2-3\left(0+i\frac{4}{n}\right)\right)\frac{4}{n} = \lim_{n\to\infty}\left(\frac{8}{n}\sum_{i=1}^{n} 1 - \frac{48}{n^2}\sum_{i=1}^{n} i\right) = \lim_{n\to\infty}\left(8 - \frac{48}{n^2}\cdot\frac{n(n+1)}{2}\right)$

$= 8 - 24 = \boxed{-16}$