MATH 220: CALCULUS I

WORKSHEET 9

FEBRUARY 14, 2013

List of some short-cut rules:

(1) $\frac{d}{dx}(x) = 1$ (2) $\frac{d}{dx}(c) = 0$ (3) $\frac{d}{dx}(x^n) = nx^{n-1}$ (4) $\frac{d}{dx}(e^x) = e^x$

(5) $\frac{d}{dx}(cf(x)) = cf'(x)$ (6) $\frac{d}{dx}(f(x)+g(x)) = f'(x) + g'(x)$

I will use red (1-6) to indicate which rule I am using on each part at each step

Derivative = slope of the tangent line = instantaneous rate of change

Homework: In Section 3.1, do problems 3-30, 33, 35, 47, 51, 53

(Hint: A *normal line* is a line perpendicular to a tangent line.)

(1) Find the equations of lines tangent to and normal to $f(x) = x^4 + 2x^2 - x$ at the point $(1, 2)$.

$$f'(x) = \frac{d}{dx}(x^4+2x^2-x) \overset{6}{=} \frac{d}{dx}(x^4) + \frac{d}{dx}(2x^2) - \frac{d}{dx}(x)$$

$$= (4x^3) + 2\cdot\frac{d}{dx}(x^2) - 1$$

$$= 4x^3 + 2\cdot 2x - 1$$

$$= 4x^3 + 4x - 1$$

Slope of tangent at $x=1$ → $f'(1) = 4+4-1 = 7$ → point-slope form $y - y_1 = m(x-x_1)$

Slope of normal line: $-\frac{1}{7}$

$y - 2 = 7(x-1)$

t-line: $y = 7x - 5$

normal-line: $y = -\frac{1}{7}(x-1)+2$

(2) Where does the graph $y = \frac{3x^4}{4} + x^3 - 3x^2 + 5$ have horizontal tangent lines?

slope of t-line = 0

$$y' \overset{6}{=} \frac{d}{dx}\left(\frac{3x^4}{4}\right) + \frac{d}{dx}(x^3) - \frac{d}{dx}(3x^2) + \frac{d}{dx}(5)$$

$$= \frac{3}{4}(x^4)' + 3x^2 - 3(x^2)' + 0$$

$$= \frac{3}{4}\cdot 4x^3 + 3x^2 - 3\cdot 2x + 0$$

$= 3x^3 + 3x^2 - 6x$, so to find horz t-lines set $0 = 3x^3 + 3x^2 - 6x$

$= 3x(x^2+x-2)$

$= 3x(x+2)(x-1)$

So for $x = 0, -2, 1$ there are horizontal tangent lines

(3) Practice applying these differentiation rules to the following functions:

(a) $f(x) = \frac{5}{\sqrt[3]{x}} = 5x^{-1/3}$

$$f'(x) \overset{5}{=} 5(x^{-1/3})' \overset{3}{=} 5\cdot\left(-\tfrac{1}{3}\right)x^{-4/3} = \boxed{\frac{-5}{3x^{4/3}}}$$

(b) $f(x) = 3e^x - 6x^4 + 10$

$$f'(x) \overset{6}{=} (3e^x)' - (6x^4)' + (10)' \overset{5}{=} 3(e^x)' - 6(x^4)' + 0$$

$$= 3e^x - 6\cdot 4x^3$$

$$= \boxed{3e^x - 24x^3}$$

(c) $g(y) = y^{-1} + \frac{y^3}{2} - \pi y + x$

$$g'(y) \overset{6}{=} (y^{-1})' + \left(\frac{y^3}{2}\right)' - (\pi y)' + (x)'$$ (taking $\frac{d}{dy}$ in this case so $\frac{d}{dy}(x) = 0$)

$$= -y^{-2} + \frac{1}{2}(y^3)' - \pi(y)' + 0$$

$$= -y^{-2} + \frac{1}{2}\cdot 3y^2 - \pi\cdot 1 = \boxed{-y^{-2} + \frac{3}{2}y^2 - \pi}$$

(d) $f(x) = (1-\sqrt{x})(x^{3/2} + 5x^2)$

$$= x^{3/2} + 5x^2 - x^{4/2} - 5x^{5/2}$$

$$f'(x) = (x^{3/2})' + (5x^2)' - (x^2)' - (5x^{5/2})'$$

$$= \frac{3}{2}x^{1/2} + 5(x^2)' - 2x - 5(x^{5/2})' \overset{3}{=} \frac{3}{2}x^{1/2} + 10x - 2x - 5\cdot\frac{5}{2}x^{3/2}$$

$$= \boxed{\frac{3}{2}x^{1/2} + 8x - \frac{25}{2}x^{3/2}}$$

(e) $h(t) = e^r + e^s - e^t$

$$h'(t) = 0 + 0 - e^t = \boxed{-e^t}$$ (taking $\frac{d}{dt}$, so r & s are constants)

(f) $C(r) = \pi r^2$

$$C'(r) \overset{5}{=} \pi(r^2)'$$

$$\overset{3}{=} \pi\cdot 2r$$

$$= \boxed{2\pi r}$$

(side note: derivative of a circle's area gives its circumference)