MATH 220: CALCULUS I
WORKSHEET 8
FEBRUARY 7, 2013

$\lim_{x\to 1} f(x) = f(1)$, but $f(1)$ is not defined,
so f is NOT continuous at $x=1$

1. Let $f(x) = 2x+1$ when $x \neq 1$. Is f continuous at $x = 1$?

2. (!) Find an equation of the tangent line to the curve $y = 2\sqrt{x+1}$ at the point $(3,4)$. Graph both the curve and the tangent line.

To find tangent line we need to know slope and a point.

slope of t-line = instantaneous rate of change = derivative

Use definition of derivative: $f'(3) = \lim_{h\to 0} \frac{f(3+h)-f(3)}{h} = \lim_{h\to 0} \frac{2\sqrt{3+h+1} - 2\sqrt{3+1}}{h}$

so tangent line is $\boxed{y-4 = \frac{1}{2}(x-3)}$

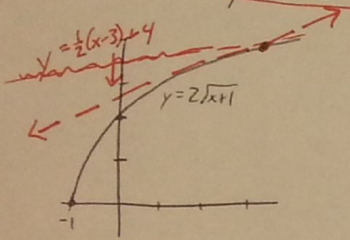
from the point $(3,4)$

rationalize numerator

$= \lim_{h\to 0} \frac{2\sqrt{4+h}-4}{h} \cdot \frac{2\sqrt{4+h}+4}{2\sqrt{4+h}+4} = \lim_{h\to 0} \frac{4(4+h)-16}{h(2\sqrt{4+h}+4)}$

plug in

$= \lim_{h\to 0} \frac{4h}{h(2\sqrt{4+h}+4)} = \frac{4}{2\sqrt{4+0}+4} = \frac{1}{2}$ ← slope of tangent line.



3. What is the slope of the line tangent to $y = \sin(x)$ at $x = 0$?

use defn of derivative:

$f'(0) = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0} \frac{\sin(0+h) - \overset{0}{\sin(0)}}{h}$

shown in class

$= \lim_{h\to 0} \frac{\sin(h)}{h} = \boxed{1}$

4. Given $f(x) = x^{-3}$, find $f'(a)$.

earlier I used $f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$, but it is also valid to use $f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$, so for variety, I'll use the second version here

$f'(a) = \lim_{x\to a} \frac{\frac{1}{x^3} - \frac{1}{a^3}}{x-a}$

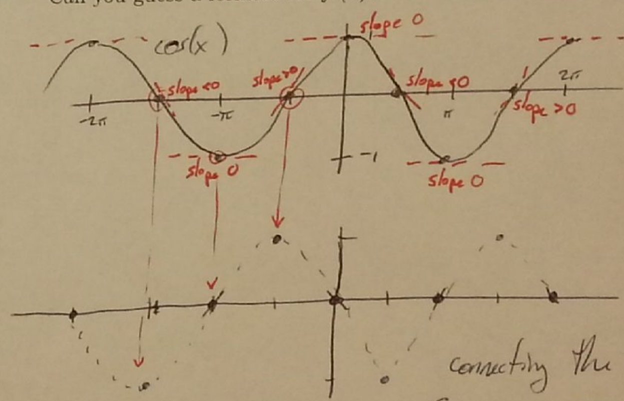
plug in

$= \lim_{x\to a} \frac{\frac{a^3-x^3}{x^3a^3}}{x-a} = \lim_{x\to a} \frac{a^3-x^3}{(x-a)(x^3a^3)} = \lim_{x\to a} -\frac{a^2+ax+x^2}{x^3a^3} = -\frac{a^2+a\cdot a+a^2}{a^3a^3}$

$= \frac{3a^2}{a^6} = \boxed{\frac{3}{a^4}}$

by polynomial long division,
$\frac{a^3-x^3}{x-a} = -(a^2+ax+x^2)$

5. Graph $f(x) = \cos(x)$. From its graph, sketch a graph of $f'(x)$. Can you guess a formula for $f'(x)$?



at each point, check the slope and plot it below

• each of the slopes that are diagonal look to be about ± 1

connecting the dots makes it look like $-\sin(x)$

So we may guess $f'(x) = -\sin(x)$

6. True or False: A function that is continuous at a point a must also be differentiable at a.

7. True or False: A function that is differentiable at a point a must also be continuous at a.