MATH 220: CALCULUS I
WORKSHEET 7
FEBRUARY 5, 2013

1. Without looking at your notes, state the condition for a function f to be *continuous* at a point a?

$$\boxed{\lim_{x\to a} f(x) = f(a)}$$

"limit equals the actual function value"

2. Let $f(x) = \begin{cases} 1+c\cdot\cos(x), & \text{if } x < \pi \\ \sin(x) - 2c, & \text{if } x \geq \pi \end{cases}$. What value of c makes f continuous everywhere?

$\lim_{x\to\pi} f(x) = f(\pi) = -2c$

2 pieces, so we look at left- & right-hand limits.

$\lim_{x\to\pi^-} f(x) = \lim_{x\to\pi^-} (1+c\cdot\cos(x)) = 1-c$

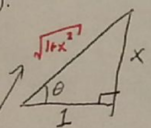
$\lim_{x\to\pi^+} f(x) = \lim_{x\to\pi^+} (\sin(x) - 2c) = -2c$

these must match, so $1-c = -2c$

$$\boxed{c = -1}$$

3. Simplify $\sin(\arctan(x))$ and $\cos(\tan^{-1}(x))$.

draw triangle



$\tan\theta = \frac{x}{1}$

$\tan^{-1}(x) = \theta$

find from Pyth.'s theorem,
Now finding any trig function with respect to θ is easy.

$\sin(\arctan(x)) = \sin\theta = \boxed{\frac{x}{\sqrt{1+x^2}}}$

$\cos(\tan^{-1}(x)) = \cos\theta = \boxed{\frac{1}{\sqrt{1+x^2}}}$

4. Is $f(x) = \begin{cases} \sin(x), & \text{if } x \neq \pi \\ 1, & \text{if } x = \pi \end{cases}$ continuous at $x = \pi$?

$\lim_{x\to\pi} f(x) = f(\pi)$

$\lim_{x\to\pi} f(x) = \lim_{x\to\pi} \sin(x) = \sin(\pi) = 0 \neq f(\pi) = 1$

So f is $\boxed{\text{NOT}}$ continuous at $x = \pi$

5. Calculate the following limit: $\lim_{x\to\infty} \frac{2x^2+1}{\sqrt{x^4 - x + 2}}$.

→ try "plugging in" & get $\frac{\infty}{\infty}$, so pull out highest powers.

$\frac{2x^2+1}{\sqrt{x^4-x+2}} = \frac{x^2(2+\frac{1}{x^2})}{\sqrt{x^4(1-\frac{x}{x^4}+\frac{2}{x^4})}} = \frac{x^2(2+\frac{1}{x^2})}{\sqrt{x^4}\sqrt{1-\frac{1}{x^3}+\frac{2}{x^4}}} = \frac{2+\frac{1}{x^2}}{\sqrt{1-\frac{1}{x^3}+\frac{2}{x^4}}}$

So $\lim_{x\to\infty} \frac{2x^2+1}{\sqrt{x^4-x+2}} = \lim_{x\to\infty} \frac{2+\frac{1}{x^2}}{\sqrt{1-\frac{1}{x^3}+\frac{2}{x^4}}} = \frac{2+0}{\sqrt{1-0+0}} = \boxed{2}$

try plugging in again

6. Formally state the Intermediate Value Theorem (be precise).

If f is continuous on $[a,b]$ & c is between $f(a)$ and $f(b)$, then some $d \in [a,b]$ satisfies $f(d) = c$.

7. Intuitively, what does the IVT *mean*?

A continuous function must hit all outputs (y-vals) between the starting ($f(a)$) and ending points. ($f(b)$)

8. State (but do not solve) one application of the IVT.

Show there is a root of $f(x)$ on the interval (a,b)
(where f is negative & then positive in (a,b), for example)