MATH 220: CALCULUS I
WORKSHEET 5 SOLUTIONS
JANUARY 29, 2013

1. For each of the following conditions, give two functions that satisfy them–a one-to-one function and a function that is not one-to-one. If no such function exists, state why.
I'll abbreviate one-to-one with 1-1

   (a) $f(1) = 2$:
   1-1: $f(x) = x+1$ or $f(x) = 2x$ work. Both are invertible and pass horizontal line test
   Not 1-1: $f(x) = x^2 + 1$ since $f(1) = 2$ and $f(-1) = 2$, so it fails horizontal line test

   (b) f is an even function:
   1-1: Can't happen! Take any x, say $x = 4$. Then clearly $4 \neq -4$, but $f(4) = f(-4)$ since f is even, and this is exactly what can't happen for a function to be 1-1.
   Not 1-1: Any even function! $f(x) = x^2$, $f(x) = \cos x$, etc.

   (c) f is an odd function:
   1-1: $f(x) = x$ or $f(x) = x^3$ are both odd and invertible.
   Not 1-1: $f(x) = \sin x$ *badly* fails the horizontal line test.

2. Find the inverse function for $f(x) = \dfrac{e^x}{2+3e^x}$ **(typo in original had e^{2x})**
The first goal is to solve the equation for x, but since x is in the exponent, the goal is to solve for e^x. Then we take the logarithm of both sides. We begin by switching the roles of x and y.

$$
\begin{aligned}
x &= \frac{e^y}{2+3e^y} \\
x(2+3e^y) &= e^y \\
2x + 3xe^y &= e^y \\
2x &= e^y - 3xe^y \\
2x &= e^y(1-3x) \\
\frac{2x}{1-3x} &= e^y \\
\ln\left(\frac{2x}{1-3x}\right) &= y = f^{-1}(x)
\end{aligned}
$$

3. What is the domain of $f(x) = \ln(e^x - 2) - 1$.
What is the domain of $f^{-1}(x)$?
a) $f(x)$ is defined only when $e^x - 2 > 0$ because logarithms are only defined on positve numbers. So we must have $e^x > 2$, which simplifies to $x > \ln 2$, and this is the domain.

b) The domain of $f^{-1}(x)$ is the range of $f(x)$ **if and only if** $f^{-1}(x)$ exists. We find $f^{-1}(x)$ by switching variable names: $x = \ln(e^y - 2) - 1$, so $x + 1 = \ln(e^y - 2)$ and $e^{x+1} = e^y - 2$. Continuing, $e^{x+1} + 2 = e^y$, so $f^{-1}(x) = y = \ln(e^{x+1} + 2)$. As before, we need $e^{x+1} + 2 > 0$, but since e^{x+1} is always positive, the inequality is **always** true. Thus the domain of $f^{-1}(x)$ is $(-\infty, \infty)$, all reals, or $\mathbb{R}$.