MATH 220: CALCULUS I
WORKSHEET 4
JANUARY 24, 2013

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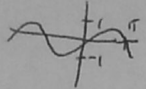
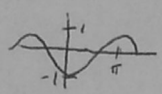
1. Sketch the graph of $2\sin(x - \frac{\pi}{2}) + 2$.

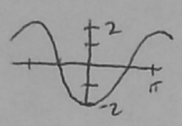
$\sin(x)$:



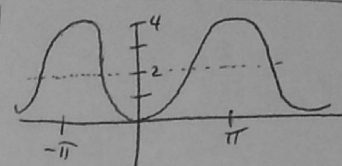
$\sin(x-\frac{\pi}{2})$:



$2\sin(x-\frac{\pi}{2})$:

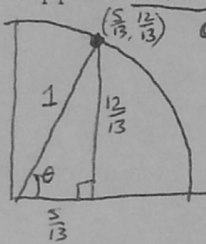


$2\sin(x-\frac{\pi}{2})+2$:



2. Suppose $0 < \theta < \pi/2$ and $\cos(\theta) = \frac{5}{13}$. Compute $\cos(\frac{\pi}{2} - \theta)$.

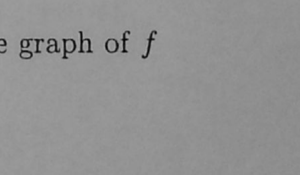
Quadrant 1



Since $\cos(\frac{\pi}{2}-\theta)=\sin(\theta)$ we can use the triangle to see that $\boxed{\sin(\theta)=\frac{12}{13}}$

$\cos(\frac{\pi}{2}-\theta)$

input	output
$\theta=0$	$\cos(\frac{\pi}{2})=0$
$\theta=\frac{\pi}{2}$	$\cos(0)=1$
$\theta=-\frac{\pi}{2}$	$\cos(\pi)=-1$
$\theta=\pi$	$\cos(\frac{\pi}{2})=0$



" $\sin(\theta)$

3. Find the function $f(x) = ka^x$ such that the points $(1, 2)$ and $(3, 16)$ lie on the graph of f where the domain of f is the entire real line.

Since $(1,2)$ is on the curve, $f(1)=2$, so $ka^1=2$
Similarly $(3,16)$ means $ka^3=16$

$ka=2 \rightarrow k=\frac{2}{a}$, so $ka^3=(\frac{2}{a})a^3=16 \rightarrow 2a^2=8$ & $a=\pm\sqrt{8}=\pm 2\sqrt{2}$

But if a is negative, then $x=\frac{1}{2}$ means taking a square root of a negative number & since the domain of f is all real numbers, a must be positive, so $\underline{a=2\sqrt{2}}$

So $ka=2$ means $k(2\sqrt{2})=2 \rightarrow \underline{k=\frac{1}{\sqrt{2}}}$

$$\boxed{f(x)=\frac{1}{\sqrt{2}}(2\sqrt{2})^x}$$

4. If $f(x) = 2^x$, show that $\dfrac{f(x+h) - f(x)}{h} = 2^x\left(\dfrac{2^h - 1}{h}\right)$.

$f(x+h) = 2^{x+h}$, so $\dfrac{f(x+h)-f(x)}{h} = \dfrac{2^{x+h}-2^x}{h} = \dfrac{2^x \cdot 2^h - 2^x}{h} = 2^x\left(\dfrac{2^h-1}{h}\right)$,

which is what we wanted.

5. Determine the exact value for each solution to the equation $\ln(4-x) + \ln(4+x) = 0$.

$\ln(4+x)+\ln(4-x) = \ln((4+x)(4-x)) = \ln(16-x^2)$

So $\ln(16-x^2)=0$, & we exponentiate both sides: $e^{\ln(16-x^2)} = e^0$

So $16-x^2=1$ & $\boxed{x=\pm\sqrt{15}}$

Note that since $\sqrt{15}<4$ both are in the domain of $\ln(4\pm x)$ (need to avoid taking logs of negative numbers)

6. A bacterial culture starts with 100 bacteria and quadruples in size every 2 hours.

(a) Find a formula for the number of bacteria as a function of the number of hours since its population was 100.

$\boxed{f(x) = 100 \cdot 4^{x/2}}$ every two hours

start quadruples...

$4^{x/2} = (4^{1/2})^x = 2^x$,

So $f(x) = 100 \cdot 2^x$ also works

(b) At what time is the population equal to 1000?

Set $f(x)=1000$ and solve for x

$1000 = 100 \cdot 2^x$

$10 = 2^x$

take a log of both sides, base doesn't matter, so I'll show 3 ways, all give same answer if you plug into a calculator

$\ln 10 = \ln 2^x \rightarrow \boxed{x = \dfrac{\ln 10}{\ln 2}}$

$\log 10 = \log 2^x \rightarrow \boxed{x = \dfrac{1}{\log 2}}$

$\log_2 10 = \log_2 2^x \rightarrow \boxed{x = \log_2 10}$