

MATH 220: CALCULUS I
 WORKSHEET 26
 APRIL 25, 2013

Homework: Section 3.11, Problems 1, 2, 3, 4, 23abcd, 31, 32, 33, 35, 38

1. Find the numerical value of any four of the following expressions (your choice).

- $\sinh(0)$
- $\tanh(0)$
- $\sinh(\ln 2)$
- $\cosh(3)$
- $\cosh(0)$
- $\tanh(1)$
- $\sinh(2)$
- $\cosh(\ln 3)$

$$\sinh(0) = \frac{e^0 - e^{-0}}{2} = 0$$

$$\cosh(0) = \frac{e^0 + e^{-0}}{2} = 1$$

$$\tanh(0) = \frac{\sinh(0)}{\cosh(0)} = 0$$

$$\tanh(1) = \frac{\sinh(1)}{\cosh(1)} = \frac{\frac{1}{2}(e^1 - e^{-1})}{\frac{1}{2}(e^1 + e^{-1})}$$

$$\sinh(\ln 2) = \frac{e^{\ln 2} - e^{-\ln 2}}{2} = \frac{2 - \frac{1}{2}}{2} = \frac{2 - \frac{1}{2}}{2} = \frac{3}{4}$$

$$\cosh(3) = \frac{e^3 + e^{-3}}{2}$$

$$\cosh(\ln 3) = \frac{e^{\ln 3} + e^{-\ln 3}}{2} = \frac{3 + \frac{1}{3}}{2} = \frac{3 + \frac{1}{3}}{2} = \frac{5}{3}$$

$$\sinh(2) = \frac{e^2 - e^{-2}}{2}$$

2. Evaluate two of the following four limits (your choice).

- $\lim_{x \rightarrow \infty} \tanh(x)$
- $\lim_{x \rightarrow -\infty} \tanh(x)$
- $\lim_{x \rightarrow \infty} \sinh(x)$
- $\lim_{x \rightarrow -\infty} \sinh(x)$

$$\lim_{x \rightarrow \infty} \tanh(x) = \lim_{x \rightarrow \infty} \frac{\sinh(x)}{\cosh(x)} = \lim_{x \rightarrow \infty} \frac{\frac{1}{2}(e^x - e^{-x})}{\frac{1}{2}(e^x + e^{-x})} = \lim_{x \rightarrow \infty} \frac{e^x - e^{-x}}{e^x + e^{-x}} = \lim_{x \rightarrow \infty} \frac{\frac{1}{e^x}(e^x - e^{-x})}{\frac{1}{e^x}(e^x + e^{-x})} = \lim_{x \rightarrow \infty} \frac{1 - e^{-2x}}{1 + e^{-2x}} = \frac{1 - 0}{1 + 0} = 1$$

$$\lim_{x \rightarrow -\infty} \tanh(x) = \lim_{x \rightarrow -\infty} \frac{e^x - e^{-x}}{e^x + e^{-x}} = \lim_{x \rightarrow -\infty} \frac{\frac{1}{e^{-x}}(e^x - e^{-x})}{\frac{1}{e^{-x}}(e^x + e^{-x})} = \lim_{x \rightarrow -\infty} \frac{e^{2x} - 1}{e^{2x} + 1} = \frac{0 - 1}{0 + 1} = -1$$

$$\lim_{x \rightarrow \infty} \sinh(x) = \lim_{x \rightarrow \infty} \frac{e^x - e^{-x}}{2} = \frac{1}{2} \lim_{x \rightarrow \infty} e^x - e^{-x} = \infty$$

$$\lim_{x \rightarrow -\infty} \sinh(x) = \lim_{x \rightarrow -\infty} \frac{e^x - e^{-x}}{2} = \frac{1}{2} \lim_{x \rightarrow -\infty} e^x - e^{-x} = -\infty$$

3. Find the derivative. Simplify where possible.

- $\sinh(\ln x)$
- $\ln(\sinh x)$
- $\cosh(\sinh x)$

$$(\sinh(\ln x))' \stackrel{CR}{=} \cosh(\ln x) \cdot (\ln x)' = \cosh(\ln x) \cdot \frac{1}{x}$$

$$(\ln(\sinh(x)))' \stackrel{CR}{=} \frac{1}{\sinh(x)} \cdot (\sinh(x))' = \frac{\cosh(x)}{\sinh(x)} = \coth(x)$$

$$(\cosh(\sinh(x)))' \stackrel{CR}{=} \sinh(\sinh(x)) \cdot (\sinh(x))' = \sinh(\sinh(x)) \cdot \cosh(x)$$