Math 220: Calculus I
Worksheet 25 - Exam 3 Super-Packet
April 23, 2013

Evaluate the following integrals (5-10 points each)

1. $\int_{-1}^{1} \left(x^2 + \sin\left(x^5\right)\right)\, dx$
2. $\int_0^2 \frac{6x^2}{x^3+1}\, dx$
3. $\int x\sqrt{2x+1}\, dx$
4. $\int \tan^3 x \sec x\, dx$
5. $\int \cos^3 x\, dx$
6. $\int_2^{18} \frac{1}{2x}\, dx$
7. $\int_0^1 \frac{8}{1+x^2}\, dx$
8. $\int \frac{12x}{1+3x^2}\, dx$
9. $\int \tan x \sec^5 x\, dx$
10. $\int 2x^5(x^2+1)^{35}\, dx$
11. $\int_{\pi/3}^{\pi/2} (12 + 6\sin x)\, dx$
12. $\int_0^2 \left(6x^2 + 3e^{-x}\right)\, dx$
13. $\int \frac{6x^3+4x^2+5x}{x^2}\, dx$
14. $\int \frac{1}{x\sqrt{\ln x}}\, dx$
15. $\int \tan^5 x \sec^4 x\, dx$
16. $\int x^2\sqrt{x+1}\, dx$
17. $\int \left(\frac{8}{x} + 4\csc^2 x + 3\right)\, dx$
18. $\int_{\pi/2}^{\pi} (10 + 3\cos x)\, dx$
19. $\int_0^2 \left(6x + 2e^{-x}\right)\, dx$
20. $\int x^3\left(x^4+7\right)^5\, dx$
21. $\int \sin^3 x \cos^5 x\, dx$
22. $\int \left(5 - 3\tan^2 x\right)\, dx$
23. $\int \left(e^x + \frac{1}{3x} + 5\right)\, dx$
24. $\int_1^2 (10x+5)\, dx$
25. $\int_0^2 \left(3 + 2e^{-x}\right)\, dx$
26. $\int x^2\sqrt{x^3+4}\, dx$
27. $\int x^2(x+4)^{10}\, dx$
28. $\int \sec^6 x \tan^3 x\, dx$

Riemann Sums

1. (8 points each) Fill in the missing information to show that each given definite integral can be expressed as the limit of a Riemann sum. The only variables appearing in your limit should be n and k. You do not need to evaluate these limits.
   - $\int_2^6 \sin\left(x^2\right)\, dx = \lim_{n\to\infty} \sum_{k=1}^{n} \Big[\qquad \Big]$
   - $\int_2^6 \left(x^5+8\right)^4\, dx = \lim_{n\to\infty} \sum_{k=1}^{n} \Big[\qquad \Big]$
   - $\int_2^6 e^{t^2}\, dt = \lim_{n\to\infty} \sum_{k=1}^{n} \Big[\qquad \Big]$
2. (6-8 points each) Using proper notation, evaluate the following limits. Simplify your answer.
   - $\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{17}{4n} - \frac{5k}{2n^2}\right)$
   - $\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{5k}{n^3} + \frac{7}{n}\right)$
   - $\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{14k}{n^2} - \frac{4}{n}\right)$

Antiderivatives and Areas

1. (10 points) Determine the formula for a function $f(x)$ such that $f''(x) = 12e^{2x} + \cos x$, $f'(0) = 10$ and $f(0) = 8$.

2. (6 points) A function $f(x)$ has derivative $f'(x) = 6x^2 + 5$. Find a formula for $f(x)$ given that its graph goes through the point $(1, 15)$.

3. (6 points) The height of a tree is currently 100 inches. It is predicted that over the next 4 years the tree's height will increase by $10 - 3\sqrt{t}$ inches per year where t represents the number of years from now. What will the tree's height be 4 years from now? Simplify your answer.

4. (6 points) Suppose $F(x)$ is a polynomial with $F'(x) = f(x)$. Given that $F(0) = 2$, $F(2) = 8$, $F(4) = 28$, $F(6) = 68$ and $F(8) = 42$, find the average value of $f(x)$ on the interval $[2, 6]$.

5. (8 points) Find a formula for $f(x)$ given that $f''(x) = 5\sin x + 3\cos x$, $f(0) = 10$, and $f'(0) = 10$.

6. (6 points) The population of a town is currently 400, but it is expected to increase at a rate of $200e^{0.5t}$ people per year where t represents the number of years from now. What is the population of this town expected to be in 10 years?

7. Find the average value of $f(x) = \dfrac{e^{3x}}{\pi - 2}$ on the interval $[3, \pi]$.

Integral Properties

1. (15 points) You are given the following definite integrals of an odd function $f(x)$.

   $\int_0^5 f(x)\,dx = 10$ $\int_0^8 f(x)\,dx = 22$ $\int_2^8 f(x)\,dx = 16$

   Evaluate the following definite integrals.

   (a) $\int_8^8 \cos(f(x))\,dx$

   (b) $\int_8^2 10f(x)\,dx$

   (c) $\int_{-2}^8 (f(x) + 5)\,dx$

   (d) $\int_2^5 f(x)\,dx$

   (e) $\int_0^{\sqrt{5}} 6xf(x^2)\,dx$

2. (3 points each) Suppose that f is an odd function which is integrable on the interval $[-5, 5]$. If $\int_0^2 f(x)\,dx = 4$ and $\int_2^3 f(x)\,dx = 10$, then evaluate the following quantities.

   (a) $\int_0^5 f(x)\,dx + \int_5^3 f(x)\,dx$ (b) $\int_{-2}^2 f(x)\,dx$ (c) $\int_{-2}^2 f(|x|)\,dx$

3. (5 points) Suppose that F and F' are each differentiable (and thus continuous) everywhere and that r and s are constants. Circle the choice below which most clearly states part 2 of the Fundamental Theorem of Calculus.

(a) $\displaystyle\int_r^s F'(t)\,dt = F'(r) - F'(s)$

(b) $\displaystyle\int_r^s F(t)\,dt = F'(r) - F'(s)$

(c) $\displaystyle\int_r^s F'(t)\,dt = F(r) - F(s)$

(d) $\displaystyle\int_r^s F(t)\,dt = F(r) - F(s)$

(e) $\displaystyle\int_r^s F'(t)\,dt = F'(s) - F'(r)$

(f) $\displaystyle\int_r^s F(t)\,dt = F'(s) - F'(r)$

(g) $\displaystyle\int_r^s F'(t)\,dt = F(s) - F(r)$

(h) $\displaystyle\int_r^s F(t)\,dt = F(s) - F(r)$

4. (12 points) Suppose f is an even function, g is an odd function, and f and g are each integrable on the interval $[-3, 3]$. Given that $\displaystyle\int_0^3 f(x)\,dx = 5$ and $\displaystyle\int_0^3 g(x)\,dx = 4$, evaluate the following definite integrals.

(a) $\displaystyle\int_3^0 g(x)\,dx$

(b) $\displaystyle\int_3^3 f(x)\,dx$

(c) $\displaystyle\int_{-3}^3 (2f(x) + 4g(x))\,dx$

(d) $\displaystyle\int_{-3}^3 \left(4 + (g(x))^5\right)\,dx$

5. (4 points each) Suppose that f is integrable on the interval $[2, 12]$. Given that $\displaystyle\int_2^{12} f(x)\,dx = 25$, $\displaystyle\int_2^8 f(x)\,dx = 10$ and $\displaystyle\int_4^{12} f(x)\,dx = 22$, evaluate the following definite integrals.

(a) $\displaystyle\int_8^2 f(x)\,dx$

(b) $\displaystyle\int_2^4 f(x)\,dx$

(c) $\displaystyle\int_4^8 f(x)\,dx$

6. (12 points) Suppose that f is an odd function and g is even which are each integrable on the interval $[-5, 5]$. Given that $\displaystyle\int_0^5 f(x)\,dx = 8$ and $\displaystyle\int_0^5 g(x)\,dx = 3$, evaluate the following definite integrals.

(a) $\displaystyle\int_5^0 g(x)\,dx$

(b) $\displaystyle\int_5^5 f(x)\,dx$

(c) $\displaystyle\int_{-5}^5 (2f(x) + 4g(x))\,dx$

(d) $\displaystyle\int_{-5}^5 \left(4 + (f(x))^3\right)\,dx$

Linear Approximation and Newton's Method

1. (7 points) Suppose that a polynomial g satisfies the following conditions.

- $g(2) = 5$
- $g'(2) = 3$
- $g''(2) = 4$
- $g'''(2) = 1$

Use a linear approximation to estimate the value of $g(1.9)$. Write your answer in decimal form.

2. (5 points) If Newton's Method is used to approximate a solution to the equation $f(x) = 0$, then it generates a sequence of approximations $x_1, x_2, x_3, x_4, \ldots$. Which one of the following correctly shows how x_n can be used to determine the next approximation x_{n+1} ?

(a) $x_{n+1} = \dfrac{x_n + f'(x_n)}{f(x_n)}$

(b) $x_{n+1} = x_n + \dfrac{f'(x_n)}{f(x_n)}$

(c) $x_{n+1} = \dfrac{x_n + f(x_n)}{f'(x_n)}$

(d) $x_{n+1} = x_n + \dfrac{f(x_n)}{f'(x_n)}$

(e) $x_{n+1} = \dfrac{x_n - f'(x_n)}{f(x_n)}$

(f) $x_{n+1} = x_n - \dfrac{f'(x_n)}{f(x_n)}$

(g) $x_{n+1} = \dfrac{x_n - f(x_n)}{f'(x_n)}$

(h) $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

3. (6 points) Suppose that a polynomial f satisfies the following conditions.

   - $f(1) = 8$
   - $f'(1) = 2$
   - $f''(1) = 3$
   - $f'''(1) = 5$

   Use a linear approximation to estimate the value of $f(0.8)$. Write your answer in decimal form.

4. (8 points) Determine an appropriate linear approximation of the function $f(x) = \sqrt{x}$ and use it to approximate $\sqrt{26.3}$. Write your answer in decimal form.

5. (6 points) The function $f(x) = 10x^3 - 20x + 1$ has one root in the interval $[1, 2]$. In order to approximate this root, begin with an initial estimate of $x_1 = 2$ and use Newton's Method to obtain a second estimate x_2. Write the value of x_2 in decimal form.

Volumes

1. (10 points) Let **R** be the finite region bounded by $8y = x^2$ and $x = y^2$. In the following manner set up, but do not evaluate, definite integrals which represent the volume of the solid obtained when **R** is revolved around the vertical line $x = 10$.

   (a) Integrate with respect to x.

   (b) Integrate with respect to y. (The integrands in parts (a) and (b) should be different.)

2. (5 points each) Let **R** be the finite region bounded by the graph of $f(x) = 5x - x^2$ and the x-axis on the interval $[0, 5]$. Set up, but do not evaluate, definite integrals which represent the given quantities. Use proper notation.

   (a) The average value of f on the interval $[0, 5]$.

   (b) The area of **R**.

   (c) The volume of the solid obtained when **R** is revolved around the horizontal line $y = -10$.

   (d) The volume of the solid obtained when **R** is revolved around the vertical line $x = 8$.

3. (6 points each) The intersection points on the graphs of $f(x) = x^2 + 2$ and $g(x) = 3x + 6$ occur at $x = -1$ and at $x = 4$. Let **R** be the finite region bounded by the graphs of $f(x)$ and $g(x)$. Set up, but do not evaluate, definite integrals which represent the given quantities. Use proper notation.

   (a) The area of **R**.

   (b) The volume of the solid obtained when **R** is revolved around the vertical line $x = 10$.

   (c) The volume of the solid obtained when **R** is revolved around the x-axis.

4. (6 points each) Let **R** be the region bounded above by graph of $y = \dfrac{\sin x}{x}$ and bounded below by the x-axis on the interval $[2\pi, 3\pi]$. Set up, but do not evaluate, definite integrals which represent the given quantities. Use proper notation.

   (a) The area of **R**.

   (b) The volume of the solid obtained when **R** is revolved around the x-axis.

   (c) The volume of the solid obtained when **R** is revolved around the vertical line $x = 3$.