MATH 220: CALCULUS I
WORKSHEET 23
APRIL 16, 2013

1. What is the average value of $f(x) = (3 - 2x)^{-1}$ on the interval $[-1, 1]$?

Not u

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{2}\int_{-1}^{1} \frac{1}{3-2x}\,dx = \frac{1}{2}\int_5^1 \frac{1}{u}\cdot\frac{du}{-2} = -\frac{1}{4}\int_5^1 \frac{1}{u}\,du$$

$u = 3-2x$ $\quad x=-1 \to u=5$
$du = -2\,dx$ $\quad x=1 \to u=1$

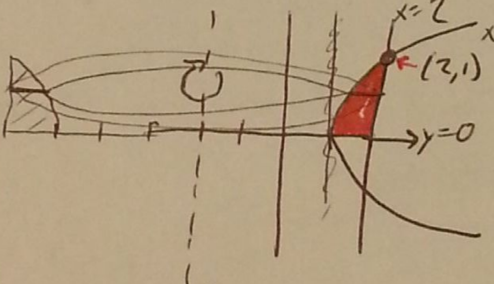
$$= -\frac{1}{4}\ln u\Big|_5^1$$

$$= -\frac{1}{4}\ln(1) - \left(-\frac{1}{4}\ln 5\right)$$

$$= \boxed{\frac{1}{4}\ln(5)}$$

2. Find the volume of the region defined by $x = y^2 + 1$, $x = 2$, and $y = 0$ when rotated about the line $x = -2$.

not 1



using washers (dy integral): $\displaystyle\int_{y=0}^{y=1} \pi(\underbrace{4}_{\text{outer radius}})^2 - \pi(\underbrace{2}_{\text{dist from y-axis to axis of rotation}} + \underbrace{(y^2+1)}_{\text{dist to y-axis}})^2\,dy = \pi\int_0^1 16 - (9 + 6y^2 + y^4)\,dy$

inner radius

$$= \pi\int_0^1 7 - 6y^2 - y^4\,dy$$

$$= \pi\left(7y - 2y^3 - \frac{1}{5}y^5\right)\Big|_0^1$$

$$= \pi\left(7 - 2 - \frac{1}{5}\right) = \boxed{\frac{24\pi}{5}}$$

using shells (dx integral): $\displaystyle\int_{x=1}^{x=2} 2\pi\underbrace{(x+2)}_{\text{radius}}\underbrace{\sqrt{x-1}}_{\text{height}}\,dx$

$$= 2\pi\int_1^2 (x+2)\sqrt{x-1}\,dx = 2\pi\int_0^1 ((u+1)+2)\sqrt{u}\,du = 2\pi\int_0^1 (u+3)\sqrt{u}\,du = 2\pi\int_0^1 u^{3/2} + 3u^{1/2}\,du$$

$x = u+1 \leftarrow u = x-1$ $\quad x=1 \to u=0$
$du = dx$ $\quad x=2 \to u=1$

$$= 2\pi\left(\frac{2}{5}u^{5/2} + 2u^{3/2}\right)\Big|_0^1 = 2\pi\left(\frac{2}{5} + 2\right) = \boxed{\frac{24\pi}{5}}$$

Either method works

$\frac{1}{\sqrt{2}}$

3. Use a linear approximation to estimate tan(44°).

Set $f(x) = \tan(x)$ so $f'(x) = \sec^2(x)$

use "nice" value for a, which in this case is $\underline{45°}$ or $\frac{\pi}{4} = a$

So the linear approximation $L(x) = f(a) + f'(a)(44° - a)$

In radians: $\frac{44°}{360°} = \frac{?}{2\pi}$ $\quad = 1 + \frac{1}{\cos^2(\frac{\pi}{4})}\left(\frac{11\pi}{45} - \frac{\pi}{4}\right)$

So $44° = \frac{11\pi}{45}$ $\quad = 1 + 2\left(-\frac{\pi}{180}\right)$

or $44° = \frac{\pi}{4} - \frac{\pi}{180}$

so that $(x-a) = \left[\left(\frac{\pi}{4} - \frac{\pi}{180}\right) - \frac{\pi}{4}\right] = -\frac{\pi}{180}$

4. Use a linear approximation to estimate the location of a root of the function $f(x) = x^3 + x^2 - 5$.

I need to pick a "nice" starting point.
There isn't "one" correct starting point, but $a = 1$ works.
Since I want a root, what I'm searching for is a point where $f(x) = 0$
and of some other candidates for "nice" values (say $a = -1, 0, 2, 3, \ldots$)
$a = 1$ has the closest output to zero

$f'(x) = 3x^2 + 2x$

So now $L(x) = f(a) + f'(a)(x-a)$

$= f(1) + f'(1)(x-1)$

$= -3 + 5(x-1)$

$= \underline{5x - 8}$

SIDE NOTE
Looking ahead to Newton's method, setting $x_1 = a = 1$, we find x_2 by solving $L(x) = 0$, giving us $x_2 = \frac{8}{5}$