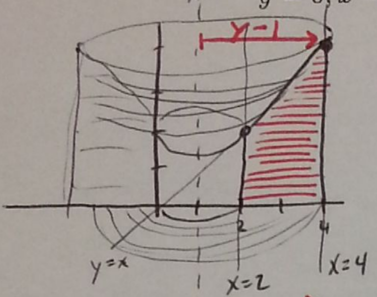
MATH 220: CALCULUS I
WORKSHEET 22
APRIL 11, 2013

1. Find the volume of the solid obtained by rotating the region defined by $y = x$, $y = 0$, $x = 2$, and $x = 4$ about the line $x = 1$.



$R=3$

$r=1$ or $r=y-1$

Start from $y=0$ & go to $y=4$, but because the "left" function is different above & below $y=2$, we must split the integral.

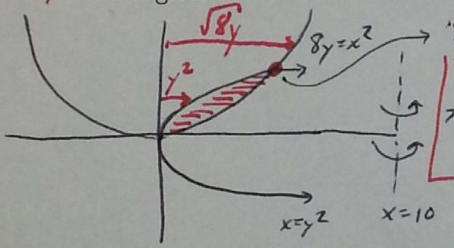
$$\int_{y=0}^{y=2} \pi 3^2 - \pi 1^2\,dy + \int_{y=2}^{y=4} \pi 3^2 - \pi (y-1)^2\,dy$$

$$= \pi(8y)\Big|_0^2 + \pi\int_2^4 9 - y^2+2y-1\,dy = 16\pi + \pi\left(8y - \tfrac{1}{3}y^3 + y^2\right)\Big|_2^4$$

$$= 16\pi + \pi\left(32 - \tfrac{64}{3} + 16\right) - \pi\left(16 - \tfrac{8}{3} + 4\right) = \boxed{\tfrac{76}{3}\pi}$$

Using "washers" → Slicing horizontally

2. Let **R** be the finite region bounded by $8y = x^2$ and $x = y^2$. Set up, **but do not evaluate**, the definite integral which represents the volume of the obtained by revolving the region **R** about the line $x = 10$. (The integral should contain a dy. Why is that?)

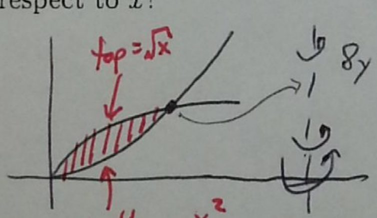


$8y=x^2,\ x=y^2 \to 8y=(y^2)^2 \to 8y=y^3 \to y=0,2$

$$\boxed{\pi\int_{y=0}^{y=2} (10-y^2)^2 - (10-\sqrt{8y})^2\,dy}$$

Using "shells" → Slicing Vertically

3. Using the region from the previous problem, how might you set up an integral with respect to x?



$8y=x^2,\ x=y^2$

$8\sqrt{x}=x^2$

$64x=x^4$

$64=x^3$

$x=4,0$

radius is the distance to axis of revolution given our position (x) in the interval

$$\int_{x=0}^{x=4} 2\pi\, r\cdot h\cdot dx$$

top - bottom

$$=\boxed{\int_0^4 2\pi(10-x)\left(\sqrt{x}-\frac{x^2}{8}\right)dx}$$