MATH 220: CALCULUS I
WORKSHEET 21
APRIL 9, 2013

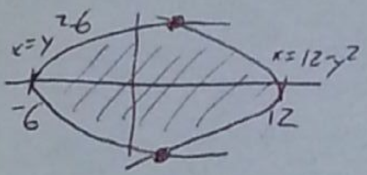
MVT IVT

1. Explain how the *Mean Value Theorem* and the *Intermediate Value Theorem* differ.

IVT: If f is continuous, then it "hits" every output value between the starting and ending output values.

MVT: If f is continuous and diff'ble, Then the instantaneous rate of change at some point is equal to the average (mean) rate of change (slope) for the whole interval.

2. Sketch the region enclosed by the curves $x = 12 - y^2$ and $x = y^2 - 6$, and find its area.



$12 - y^2 = y^2 - 6$

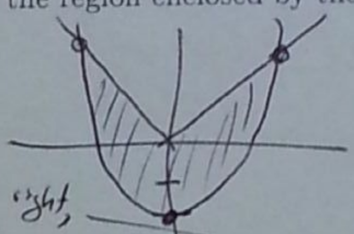
$2y^2 = 18$

$y = \pm 3$ limits of integration.

So Area $= \int_{-3}^{3}$ "right" $-$ "left" $dy = \int_{-3}^{3} (12-y^2)-(y^2-6)\,dy = \int_{-3}^{3} 18-2y^2\,dy$

By symmetry (optional), $= 2\cdot\int_0^3 18-2y^2\,dy = 2\cdot\left(18y-\frac{2}{3}y^3\right)\Big|_0^3 = 2\cdot\left(18\cdot 3-\frac{2}{3}\cdot 3^3\right) = \boxed{72}$

3. Sketch the region enclosed by the curves $y = |x|$ and $y = x^2 - 2$, and find its area.



$|x| = \begin{cases} x & \text{for } x \geq 0 \\ -x & \text{for } x < 0 \end{cases}$

So $x = x^2 - 2$ when $x \geq 0$

$\Rightarrow x^2 - x - 2 = 0$ (when $x \geq 0$)

$\Rightarrow (x-2)(x+1) = 0$

$x = 2$ (can't be -1 since $x \geq 0$)

By symmetry, the other limit is $x = -2$

(or set $-x = x^2 - 2$, which is valid for $x < 0$

$(x+2)(x-1) = 0 \rightarrow x = -2$ (

can't be $x = 1$ since $x < 0$

After doing stuff on the right,

$\int_{-2}^{2} |x| - (x^2-2)\,dx$

By symmetry, $= 2\cdot\int_0^2 |x|-(x^2-2)\,dx$

& on $[0,2]$, $|x| = x$, so

$= 2\cdot\int_0^2 x - x^2 + 2\,dx = 2\cdot\left(\frac{1}{2}x^2 - \frac{1}{3}x^3 + 2x\right)\Big|_0^2$

$= 4 - \frac{16}{3} + 8 = \boxed{\frac{20}{3}}$

FTC Part I requires a constant on bottom and something in terms of x on top. So we ~~get the~~ must split the integral.
If $g(x) = \int_a^x f(t)\,dt$, then $g'(x) = f(x)$

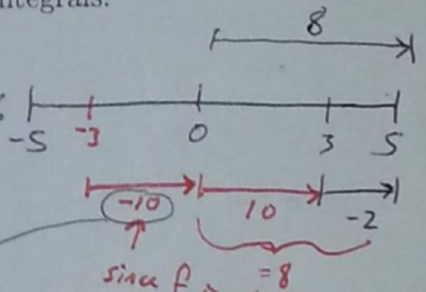
4. Given $y = \displaystyle\int_{x^2}^{x^3} \frac{1+\sin(t)}{e^{2t}}\,dt$, find $\frac{dy}{dx}$.

$$y = \int_{x^2}^{0} \frac{1+\sin(t)}{e^{2t}}\,dt + \int_0^{x^3} \frac{1+\sin(t)}{e^{2t}}\,dt = -\int_0^{x^2} \frac{1+\sin(t)}{e^{2t}}\,dt + \int_0^{x^3} \frac{1+\sin(t)}{e^{2t}}$$

$$\frac{dy}{dx} = y' = \left(-\int_0^{x^2} \frac{1+\sin(t)}{e^{2t}}\,dt\right)' + \left(\int_0^{x^3} \frac{1+\sin(t)}{e^{2t}}\,dt\right)' \overset{CR}{=} \boxed{-\frac{1+\sin(x^2)}{e^{2(x^2)}}\cdot(2x) + \frac{1+\sin(x^3)}{e^{2x^3}}\cdot(3x^2)}$$

5. (12 points) Suppose that f is an odd function and g is an even function, both of which are integrable on the interval $[-5, 5]$. Given that $\int_0^5 f(x)\,dx = 8$, $\int_3^5 f(x)\,dx = -2$, and $\int_0^5 g(x)\,dx = 3$, evaluate the following integrals.

Black lines are given, red lines we find using the given information



(a) $\displaystyle\int_5^0 2g(x)\,dx = 2\cdot\left(-\int_0^5 g(x)\,dx\right) = -2\cdot 3 = \boxed{-6}$

(b) $\displaystyle\int_{-5}^{5} 4f(x)\,dx = 0$

(c) $\displaystyle\int_0^{-3} f(x)\,dx = -\int_{-3}^{0} f(x)\,dx = \boxed{+10}$

(d) $\displaystyle\int_{-5}^{5} \left(4 + (f(x))^3\right)\,dx$

$$= \int_{-5}^{5} 4\,dx + \int_{-5}^{5} f(x)^3\,dx \underbrace{=}_{\text{Since } f \text{ is odd, } f^3 \text{ is odd, so } \int_{-5}^{5} f(x)^3\,dx = 0} 4x\Big|_{-5}^{5} + 0 = \boxed{40}$$