MATH 220: CALCULUS I
WORKSHEET 19
APRIL 2, 2013

1. If possible, evaluate $\int_0^\pi \tan x \sec x \, dx$.

Since $\tan(x)\sec(x)$ is <u>not defined</u> on the interval $[0,\pi]$, the integral <u>does not exist</u>.

($\tan(\frac{\pi}{2})\sec(\frac{\pi}{2})$ is not defined.)

2. If possible, evaluate $\int_{-1}^1 e^{2x} \, dx$.

$$\int_{-1}^1 e^{2x}\,dx = \frac{1}{2}e^{2x}\Big|_{-1}^1 = \frac{1}{2}e^{2(1)} - \frac{1}{2}e^{2(-1)} = \boxed{\frac{1}{2}(e^2 - e^{-2})}$$

<u>check</u> by taking a derivative

3. Find $\int \left(x^2 + 1 + \frac{1}{x^2+1}\right) dx$.

$$\int\left(x^2+1+\frac{1}{x^2+1}\right)dx = \int x^2\,dx + \int 1\,dx + \int \frac{1}{x^2+1}\,dx$$
$$= \boxed{\frac{1}{3}x^3 + x + \tan^{-1}(x) + C}$$

It's OK to use a single "+C" for the whole integral. You don't need one for each part.

4. If $w'(t)$ is the change in my (Tom's) weight in pounds per year, then what does $\int_{25}^{26} w'(t)\,dt$ represent?

$\int_a^b w'(t)\,dt$ is the net change in weight over the time period $[a,b]$

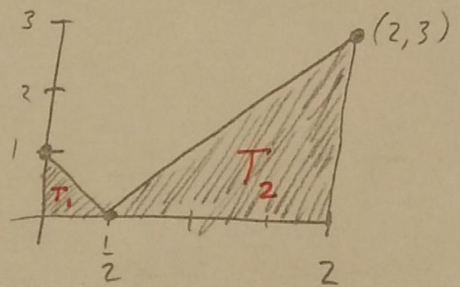
So $\int_{25}^{26} w'(t)\,dt$ is how much more or less I weighed on my 26^{th} birthday than on my 25^{th} birthday.

5. If possible, evaluate $\int_0^2 |2x-1|\,dx$.

Careful! $\int |2x-1|\,dx \neq x^2 - x + C$

Look at the graph:

$\int_0^2 |2x-1|\,dx$ is the shaded region



This one I can solve by adding the areas of the two triangles T_1, T_2

$$\underbrace{\tfrac{1}{2}\left(\tfrac{1}{2}\cdot 1\right)}_{T_1} + \underbrace{\tfrac{1}{2}\left(\tfrac{3}{2}\cdot 3\right)}_{T_2} = \frac{1}{4} + \frac{9}{4} = \frac{10}{4} = \boxed{\frac{5}{2}}$$

base height base height

OR I can split the integral up: For $0 \le x \le \frac{1}{2}$, $|2x-1| = -(2x-1)$

For $\frac{1}{2} \le x \le 2$, $|2x-1| = (2x-1)$

So $\int_0^2 |2x-1|\,dx = \int_0^{\frac{1}{2}} -(2x-1)\,dx + \int_{\frac{1}{2}}^2 (2x-1)\,dx$

$$= -x^2 + x \Big|_0^{\frac{1}{2}} + x^2 - x \Big|_{\frac{1}{2}}^2 = \left(-\left(\tfrac{1}{2}\right)^2 + \tfrac{1}{2}\right) - \left[-0^2 + 0\right] + \left(2^2 - 2\right) - \left(\left(\tfrac{1}{2}\right)^2 - \tfrac{1}{2}\right)$$

$$= -\frac{1}{4} + \frac{1}{2} + 2 - \frac{1}{4} + \frac{1}{2} = 3 - \frac{1}{2} = \boxed{\frac{5}{2}}$$

The second method is very important to know, because it does not depend on making a good picture.