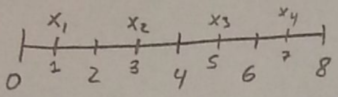
MATH 220: CALCULUS I
WORKSHEET 18
MARCH 28, 2013

1. Use the Midpoint Rule with $n = 4$ to approximate $\int_{0}^{8} (\sin\sqrt{x})\, dx$

$\int_{a}^{b} \underbrace{(\sin\sqrt{x})}_{f(x)}\, dx$

$\Delta x = \frac{8-0}{n} = \frac{8}{4} = 2$



$A \approx M_4 = f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + f(x_4)\Delta x$

$= \boxed{2(\sin\sqrt{1} + \sin\sqrt{3} + \sin\sqrt{5} + \sin\sqrt{7})}$

2. Express the limit $\lim_{n\to\infty} \sum_{i=1}^{n} \Delta x \underbrace{(3 - 4x_i)}_{f(x_i)}$ on the interval $[1, 4]$ as a definite integral.

$a = 1$, $b = 4$ $\quad \int_a^b f(x)\, dx$

$= \boxed{\int_1^4 (3-4x)\, dx}$

3. Evaluate the limit from the previous question. $\Delta x = \frac{4-1}{n} = \frac{3}{n}$

$x_i = a + (\Delta x)\cdot i = 1 + \frac{3}{n}\cdot i$

$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\left(3 - 4\left(1 + \frac{3}{n}\cdot i\right)\right)$

Goal: · Factor out anything that is NOT i
· Evaluate the limit LAST
· Split up summations if necessary

$= \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} \left(-1 - 4\cdot\frac{3}{n} i\right)$

Recall: $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

$= \lim_{n\to\infty} \left(\frac{3}{n} \sum_{i=1}^{n} (-1) - \frac{3}{n}\cdot\frac{12}{n} \sum_{i=1}^{n} i\right)$

$= \lim_{n\to\infty} \left(\frac{3}{n}\cdot(-n) - \frac{36}{n^{\cancel{2}}}\cdot\frac{\cancel{n}(n+1)}{2}\right) = \lim_{n\to\infty} \left(-3 - \frac{36}{2}\left(\frac{n+1}{n}\right)\right) = -3 - 18 = \boxed{-21}$

1 as $n \to \infty$

negative area means the curve goes below the x-axis.

4. If $\int_1^8 f(x)\, dx = 13$ and $\int_6^8 f(x)\, dx = -2$, then what is $\int_1^6 f(x)\, dx$?

Since $\int_1^8 f(x)\, dx = \int_1^6 f(x)\, dx + \int_6^8 f(x)\, dx$ ← read this line as "the area from 1 to 8 is the same as the area from 1 to 6 plus the area from 6 to 8" and it should be obvious that this line is true.

$13 = \int_1^6 f(x)\, dx - 2$

So $\int_1^6 f(x)\, dx = \boxed{15}$