MATH 220: CALCULUS I
WORKSHEET 15
MARCH 7, 2013

1. Evaluate the following limits.

(a) $\lim_{x\to 1} \dfrac{\sin(x)-1}{x^2+1}$ $\begin{matrix} \to \sin(1)-1 \\ \to 2 \end{matrix}$ $= \boxed{\dfrac{\sin(1)-1}{2}}$

0^∞ NOT indeterminate

(b) $\lim_{x\to\infty} \left(\sin\left(\dfrac{1}{x}\right)^{2x} \right) = \boxed{0}$ since something very small raised to a huge power only gets smaller

$\nearrow +\infty$ more quickly than $\ln(1-x)$

(c) $\lim_{w\to-\infty} \dfrac{e^{-w}}{\ln(1-w)} = \boxed{\infty}$

$\searrow +\infty$ more slowly than e^{-w}

$\frac{\infty}{\infty}$ is also indeterminate, so LH would work, but requires more effort than knowing that e^x grows faster than $\ln(x)$.

$\infty \cdot 0$ indeterminate form

(d) $\lim_{x\to\infty} x\tan\left(\dfrac{1}{x}\right) = \lim_{x\to\infty} \dfrac{\tan(\frac{1}{x})}{\frac{1}{x}} \overset{LH}{=} \lim_{x\to\infty} \dfrac{\sec^2(\frac{1}{x}) \cdot \frac{d}{dx}(\frac{1}{x})}{\frac{d}{dx}(\frac{1}{x})} = \lim_{x\to\infty} \dfrac{\sec^2(\frac{1}{x}) \cdot \cancel{(-\frac{1}{x^2})}}{\cancel{(-\frac{1}{x^2})}}$

need a fraction to use LH

$= \lim_{x\to\infty} \sec^2(\tfrac{1}{x}) = \sec^2(0) = \dfrac{1}{\cos^2(0)}$

$= \dfrac{1}{1^2} = \boxed{1}$

(e) $\boxed{\lim_{t\to\infty} \frac{t^{10}}{e^{t^2}} = 0}$

$t^{10} \to \infty$ slowly

$e^{t^2} \to \infty$ quickly

Since we have $\frac{\infty}{\infty}$, LH would apply, but you'd need to do it 10 times to get an answer, which would be terrible.

(f) $\lim_{x\to 0^+} x^{\sqrt[3]{x}}$

0^0 is indeterminate

$0\cdot(-\infty)$ indeterminate so LH applies

$$\lim_{x\to 0^+} x^{\sqrt[3]{x}} = e^{\lim_{x\to0^+} \ln\left(x^{\sqrt[3]{x}}\right)} = e^{\lim_{x\to0^+}\left(x^{1/3}\ln(x)\right)} = e^{\lim_{x\to0^+}\left(\frac{\ln(x)}{x^{-1/3}}\right)} \stackrel{LH}{=} e^{\lim_{x\to0^+} \frac{x^{-1}}{-\frac{1}{3}x^{-4/3}}}$$

carefully cancel exponents

$$= e^{\lim_{x\to0^+}\left(-3x^{1/3}\right)} = e^0 = \boxed{1}$$

(g) $\lim_{x\to 0} \frac{x-\sin x}{x-\tan x}$ $\quad \frac{0-0}{0-0}$

$\frac{0}{0}$ is indeterminate, so LH is allowed

$$\lim_{x\to0} \frac{x-\sin(x)}{x-\tan(x)} \stackrel{LH}{=} \lim_{x\to0} \frac{(1-\cos(x)) \to 0}{(1-\sec^2(x)) \to 0}$$

still indeterminate, so LH is allowed

$$\stackrel{LH}{=} \lim_{x\to0} \frac{0+\sin(x)}{0-2\sec(x)\underbrace{(\sec(x)\cdot\tan(x))}_{\text{chain rule on } \sec^2(x)}} = \lim_{x\to0} \frac{\sin(x)}{-2\sec^2(x)\tan(x)}$$

rewrite trig functions

$$= \lim_{x\to0} \frac{\sin(x)}{-2\frac{1}{\cos^2(x)}\frac{\sin(x)}{\cos(x)}} = \lim_{x\to0} \frac{\cos^3(x)}{-2} = \boxed{-\frac{1}{2}}$$