

1. What function y satisfies the differential equation $\frac{dy}{dx} = -x$ and goes through the point $(0, 1)$? Not y

$y = e^{-x}$ is WRONG, $\frac{dy}{dx} = -e^{-x} = -y$ in this case

So how can I be left with just $-x$ after taking a derivative?

Well, $\frac{d}{dx}(x^2) = 2x$, so $\frac{d}{dx}(-\frac{1}{2}x^2) = -x$

thus $y = -\frac{1}{2}x^2$, but $y(0)$ is not 1

However, $\frac{d}{dx}(1) = 0$, so $y = -\frac{1}{2}x^2 + 1$ still satisfies $\frac{dy}{dx} = -x$

2. Does the function $f(x) = x^9 + x^5 + x + 1$ have a local minimum or local maximum?

$$f'(x) = 9x^8 + 5x^4 + 1 > 0$$

since x^8 is never negative & x^4 is never negative,
 $f'(x) > 0$ for all values of x

Since local min/max must happen when $f'(x) = 0$,
there are NONE for this function $f(x)$

BONUS: Find intervals of concavity of $f(x)$, as well as any inflection points.

3. Let $f(x) = 5x^9 - 3x^5$. Find where f is increasing or decreasing. Find local maximum and minimum values of f . Find intervals of concavity and inflection points. Finally, sketch the graph using this information.

$$f'(x) = 45x^8 - 15x^4 = 15x^4(3x^4 - 1) = 15x^4(\sqrt{3}x^2 + 1)(\sqrt{3}x^2 - 1)$$

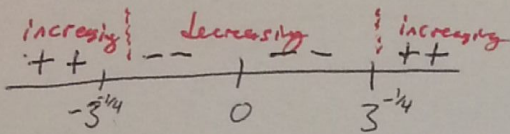
$$f'(x) = 0 \text{ when } x = 0 \text{ or } \pm 3^{-1/4}$$

these are the critical points.

$$15x^4 = 0 \rightarrow x = 0$$

$$\sqrt{3}x^2 + 1 = 0 \rightarrow \text{never happens}$$

$$\sqrt{3}x^2 - 1 = 1 \rightarrow x = \pm \frac{1}{3^{1/4}}$$

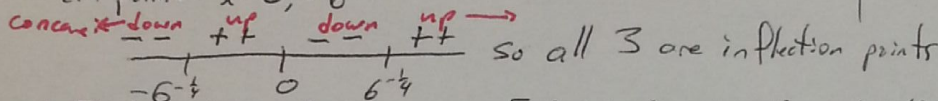


So at $x = -3^{-1/4}$ there is a local max

& at $x = 3^{-1/4}$ there is a local min

$$f''(x) = 360x^7 - 60x^3 = 60x^3(6x^4 - 1)$$

$$\text{crit points: } x = 0, \pm 6^{-1/4}$$



You could also use $3x^4 - 1 = 0$
when $x = \pm \frac{1}{3^{1/4}}$
and not use difference of squares

4. Find a point on the curve $y = \sqrt{x}$ that is closest to the point $(3, 0)$.

distance between points: ~~$d = \sqrt{(x-3)^2 + (y-0)^2}$~~

oops, my bad.

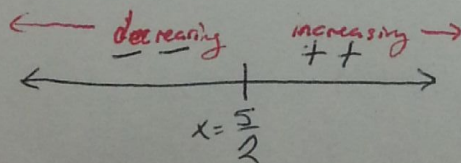
~~$$d = \sqrt{\frac{1}{2}(x^2 + (x-3)^2)}$$~~

Start here $\rightarrow d = \sqrt{(x-3)^2 + (y-0)^2} = \sqrt{x^2 - 6x + 9 + (\sqrt{x})^2} = \sqrt{x^2 - 5x + 9}$

$$d'(x) = \frac{1}{2}(x^2 - 5x + 9)^{-1/2} \cdot (2x - 5) = \frac{2x - 5}{2\sqrt{x^2 - 5x + 9}}$$

$$d'(x) = 0 \text{ when } x = \frac{5}{2} \text{ (note that when } x = \frac{5}{2}, \text{ the denominator is not zero)}$$

∇



so $x = \frac{5}{2}$ is a local min

(and absolute min since it's the only one)

Thus $(\frac{5}{2}, \sqrt{\frac{5}{2}})$ is the closest point on $y = \sqrt{x}$ to $(3, 0)$