MATH 220: CALCULUS I
WORKSHEET 14
MARCH 5, 2013

Not y

1. What function y satisfies the differential equation $\frac{dy}{dx} = -x$ and goes through the point $(0, 1)$?

$y = e^{-x}$ is WRONG. $\frac{dy}{dx} = -e^{-x} = -y$ in this case

So how can I be left with just $-x$ after taking a derivative?

Well, $\frac{d}{dx}(x^2) = 2x$, so $\frac{d}{dx}\left(-\frac{1}{2}x^2\right) = -x$

thus $y = -\frac{1}{2}x^2$, but $y(0)$ is not 1

However, $\frac{d}{dx}(1) = 0$, so $\boxed{y = -\frac{1}{2}x^2 + 1}$ still satisfies $\frac{dy}{dx} = -x$

2. Does the function $f(x) = x^9 + x^5 + x + 1$ have a local minimum or local maximum?

$f'(x) = 9x^8 + 5x^4 + 1 + 0$

since x^8 is never negative & x^4 is never negative,
$f'(x) > 0$ for all values of x

Since local min/max must happen when $f'(x) = 0$,
there are NONE for this function $f(x)$

BONUS: Find intervals of concavity of $f(x)$, as well as any inflection points.

3. Let $f(x) = 5x^9 - 3x^5$. Find where f is increasing or decreasing. Find local maximum and minimum values of f. Find intervals of concavity and inflection points. Finally, sketch the graph using this information.

$$f'(x) = 45x^8 - 15x^4 = 15x^4(3x^4-1) = 15x^4(\sqrt{3}x^2+1)(\sqrt{3}x^2-1)$$

$f'(x)=0$ when $x=0, \pm 3^{-\frac{1}{4}}$

these are the critical points.

$15x^4=0 \rightarrow x=0$

$\sqrt{3}x^2+1=0 \rightarrow$ never happens

$\sqrt{3}x^2-1=1 \rightarrow x=\pm \frac{1}{3^{1/4}}$

increasing ++ | decreasing -- | decreasing -- | increasing ++

$-3^{-1/4}$ $\quad 0 \quad$ $3^{-1/4}$

You could also use $3x^4-1=0$ when $x=\pm\frac{1}{3^{1/4}}$ and not use difference of squares

So at $x=-3^{-\frac{1}{4}}$ there is a local max

& at $x=3^{-\frac{1}{4}}$ there is a local min

$$f''(x) = 360x^7 - 60x^3 = 60x^3(6x^4-1)$$

crit points: $x=0, \pm 6^{-\frac{1}{4}}$

concave down -- | up ++ | down -- | up ++ →

$-6^{-\frac{1}{4}}$ $\quad 0 \quad$ $6^{-\frac{1}{4}}$

so all 3 are inflection points

4. Find a point on the curve $y = \sqrt{x}$ that is closest to the point $(3, 0)$.

distance between points:

oops, my bad.

Start here → $d = \sqrt{(x-3)^2+(y-0)^2} = \sqrt{x^2-6x+9+(\sqrt{x})^2} = \sqrt{x^2-5x+9}$

$$d'(x) = \frac{1}{2}(x^2-5x+9)^{-\frac{1}{2}} \cdot (2x-5) = \frac{2x-5}{2\sqrt{x^2-5x+9}}$$

$d'(x)=0$ when $x=\frac{5}{2}$ (note that when $x=\frac{5}{2}$, the denominator is not zero)

← decreasing -- | increasing ++ →

$x=\frac{5}{2}$

, so $x=\frac{5}{2}$ is a local min

(and absolute min since it's the only one)

$$\boxed{\text{Thus } \left(\underset{x}{\tfrac{5}{2}}, \underset{y=\sqrt{x}}{\sqrt{\tfrac{5}{2}}}\right) \text{ is the closest point on } y \text{ to } (3,0)}$$