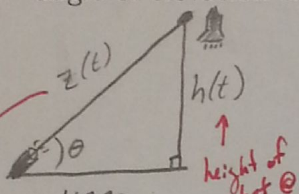
MATH 220: CALCULUS I
WORKSHEET 13
FEBRUARY 28, 2013

1. A television camera is positioned 4000 ft from the base of a rocket launching pad. The angle of elevation of the camera has to change at the correct rate in order to keep the rocket in sight. Also the mechanism for focusing the camera has to take into account the increasing distance from the camera to the rising rocket. Let's assume the rocket rises vertically and its speed is 500 ft/s when it has risen 3000 ft.

   (a) How fast is the distance from the television camera to the rocket changing at the moment? $\frac{dz}{dt}$

   (b) If the television camera is always kept aimed at the rocket, how fast is the camera's angle of elevation changing at that same moment?

a)



dist from camera to rocket @ time t

height of rocket @ time t

$z^2 = h^2 + 4000^2$

$\frac{d}{dt}(z^2) = \frac{d}{dt}(h^2 + 4000^2)$

$2z\frac{dz}{dt} = 2h\frac{dh}{dt} + 0$

quantities to plug in after taking derivative:

when $h = 3000$

$z = 5000 = \sqrt{3000^2 + 4000^2}$

$\frac{dh}{dt} = 500$ ft/s

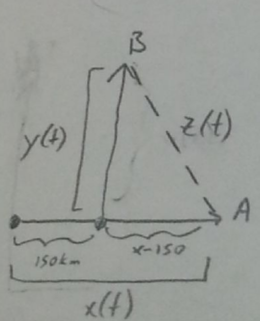
So $\frac{dz}{dt} = \frac{2\cdot 3000\cdot 500}{2\cdot 5000} = 300$ ft/s when $h(t) = 3000$

b) $\theta = \arctan\left(\frac{h(t)}{4000}\right)$ (initial relation) → take derivative

$\frac{d\theta}{dt} = \frac{1}{1+\left(\frac{h}{4000}\right)^2}\cdot\frac{1}{4000}\frac{dh}{dt} \longrightarrow \frac{d\theta}{dt} = \frac{1}{1+\left(\frac{3000}{4000}\right)^2}\cdot\frac{500}{4000} = \frac{16}{25}\cdot\frac{5}{4} = \frac{4}{5}$ radians/sec when $h(t) = 3000$

2. At noon, ship A is 150 km west of ship B. Ship A is sailing east at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 4:00 PM?

Find $\frac{dz}{dt}$



stuff to plug in later:

$\frac{dy}{dt} = 25$ km/h

$\frac{dx}{dt} = 35$ km/h

$t = 4$ hrs → $y(t) = 100$ km, $x(t) = 140$ km

$z(t) = \sqrt{100^2 + 140^2} \approx 172$ km (calculator)

$z^2 = (x-150)^2 + y^2$ ← initial relation

$\frac{d}{dt}(z^2) = \frac{d}{dt}((x-150)^2 + y^2)$ ← take derivative

$2z\frac{dz}{dt} = 2(x-150)\cdot\frac{dx}{dt} + 2y\frac{dy}{dt}$

plug in: $2\cdot 172\cdot\frac{dz}{dt} = 2(-10)\cdot 35 + 200\cdot 25$

$\frac{dz}{dt} = \frac{25}{2}$ km/h