MATH 220: CALCULUS I
WORKSHEET 12
FEBRUARY 26, 2013

1. Find $\frac{dy}{dx}$.

(a) $y = \tan^{-1}(\ln(x\sin(x)))$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\tan^{-1}(\ln(x\sin(x)))\right) = \frac{1}{1+(\ln(x\sin(x)))^2}\left(\ln(x\sin(x))\right)'$$

$$= \frac{1}{1+(\ln(x\sin x))^2}\cdot\frac{1}{x\sin x}\cdot(x\sin(x))'$$

$$= \boxed{\frac{1}{1+(\ln(x\sin(x)))^2}\cdot\frac{1}{x\sin(x)}\cdot\left(\sin(x)+x\cos(x)\right)}$$

(b) $\tan(x-y) = \dfrac{y}{1+x^2}$

$$\frac{d}{dx}(\tan(x-y)) = \frac{d}{dx}\left(\frac{y}{1+x^2}\right) \rightarrow \sec^2(x-y)\cdot\frac{d}{dx}(x-y) = \frac{(1+x^2)\cdot\frac{dy}{dx} - y\cdot 2x}{(1+x^2)^2}$$

$$\rightarrow \sec^2(x-y)\left(1-\frac{dy}{dx}\right) = \frac{(1+x^2)\frac{dy}{dx} - 2xy}{(1+x^2)^2} \rightarrow (1+x^2)^2\sec^2(x-y) - (1+x^2)^2\sec^2(x-y)\frac{dy}{dx} = (1+x^2)\frac{dy}{dx} - 2xy$$

done with derivatives, now it's just algebra

$$\rightarrow -(1+x^2)\frac{dy}{dx} - (1+x^2)^2\sec^2(x-y)\frac{dy}{dx} = -(1+x^2)^2\sec^2(x-y) - 2xy \rightarrow \frac{dy}{dx}\left(1+x^2+(1+x^2)^2\sec^2(x-y)\right) = (1+x^2)^2\sec^2(x-y) + 2xy$$

$$\rightarrow \boxed{\frac{dy}{dx} = \frac{(1+x^2)^2\sec^2(x-y)+2xy}{1+x^2+(1+x^2)^2\sec^2(x-y)}}$$

worry more about derivatives & less about algebra.

(c) $y = (x^3+1)^{\tan(x^2)}$

$$\ln y = \ln\left[(x^3+1)^{\tan(x^2)}\right] = \tan(x^2)\ln(x^3+1)$$

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}\left(\tan(x^2)\ln(x^3+1)\right)$$

$$\frac{1}{y}\frac{dy}{dx} = \left(\tan(x^2)\right)'\ln(x^3+1) + \tan(x^2)\left(\ln(x^3+1)\right)'$$

$$\frac{dy}{dx} = y\left(\sec^2(x^2)\cdot 2x\cdot\ln(x^3+1) + \tan(x^2)\cdot\frac{3x^2}{x^3+1}\right)$$

$$\boxed{\frac{dy}{dx} = (x^3+1)^{\tan(x^2)}\left(2x\sec^2(x^2)\ln(x^3+1) + \frac{3x^2}{x^3+1}\tan(x^2)\right)}$$

2. Using implicit differentiation, find $\frac{d}{dx}\left(\cos^{-1}(x)\right)$.

let $y = \cos^{-1}(x)$, so $\cos y = x$.

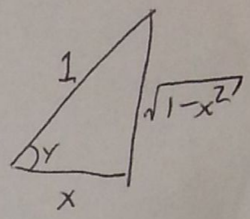
$\frac{d}{dx}(\cos y) = \frac{d}{dx}(x)$

$-\sin(y) \cdot \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{-1}{\sin(y)}$ two ways to find sin(y)

$$\boxed{\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}}$$

1) Draw triangle



so $\sin(y) = \sqrt{1-x^2}$

2) Use identities: $\cos^2 y + \sin^2(y) = 1$

so $\sin(y) = \sqrt{1-\cos^2(y)}$

$= \sqrt{1-x^2}$

3. Find $\frac{dy}{dx}$ for $x^y = y^x$.

$\ln(x^y) = \ln(y^x) \rightarrow y\ln(x) = x\ln(y)$

$\frac{d}{dx}(y\ln(x)) = \frac{d}{dx}(x\ln y)$

$\frac{dy}{dx}\ln(x) + \frac{y}{x} = \ln y + \frac{x}{y}\cdot\frac{dy}{dx}$

$\frac{dy}{dx}\ln(x) - \frac{x}{y}\frac{dy}{dx} = \ln y - \frac{y}{x}$

$\frac{dy}{dx}\left(\ln x - \frac{x}{y}\right) = \ln(y) - \frac{y}{x}$

$$\boxed{\frac{dy}{dx} = \frac{\ln y - \frac{y}{x}}{\ln x - \frac{x}{y}}}$$