MATH 220: CALCULUS I
WORKSHEET ~~#~~ 11
FEBRUARY 21, 2013

List of some short-cut rules:

$\frac{d}{dx}(x) = 1$	$\frac{d}{dx}(c) = 0$	$\frac{d}{dx}(x^n) = nx^{n-1}$	$\frac{d}{dx}(e^x) = e^x$
$\frac{d}{dx}(a^x) = a^x \ln(a)$	$\frac{d}{dx}(\ln x) = \frac{1}{x}$	$\frac{d}{dx}(\sin x) = \cos x$	$\frac{d}{dx}(\cos x) = -\sin x$
$\frac{d}{dx}(\tan(x)) = \sec^2(x)$	$\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x)$	$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$	$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(cf(x)) = cf'(x)$

$\frac{d}{dx}(f(x) + g(x)) = f'(x) + g'(x)$

$\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$

$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

$\frac{d}{dx}(\ f(g(x))\ ) = f'(\ g(x)\ ) \cdot g'(x)$

Derivative = slope of the tangent line = instantaneous rate of change

Homework: 3.1: 3-30, 33, 35, 47, 51, 53
3.2: 3-33 (odd only)
3.3: 1-23 (odd only)
3.4: 7-55 (odd only)
Total of $28 + 5 + 16 + 12 + 25 = 86$ exercises.

1. Differentiate.

   (a) $y = e^{3\ln(e^{\sin x})} = e^{3\sin(x)\cdot \ln(e)} = e^{3\sin(x)}$

   $$\frac{dy}{dx} = y' \overset{CR}{=} e^{3\sin(x)} \cdot (3\sin(x))' = e^{3\sin(x)} \cdot 3\cos(x)$$
   $$= \boxed{3\cos(x)e^{3\sin(x)}}$$

   (b) $f(t) = \tan(e^{2t})$

   $$\frac{d}{dt}f(t) = f'(t) \overset{CR}{=} \sec^2(e^{2t}) \cdot (e^{2t})' \overset{CR}{=} \sec^2(e^{2t}) \cdot e^{2t}(2t)'$$
   $$= \sec^2(e^{2t})\, e^{2t} \cdot 2$$
   $$= \boxed{2\sec^2(e^{2t})e^{2t})}$$

(c) $g(x) = \left(\dfrac{\sin(x)}{e^x + e^1}\right)^2$

$$\frac{d}{dx}g(x) = g'(x) \overset{CR}{=} 2\left(\frac{\sin(x)}{e^x+e^1}\right)\cdot\left(\frac{\sin(x)}{e^x+e^1}\right)'$$

$$\overset{QR}{=} 2\left(\frac{\sin(x)}{e^x+e^1}\right)\cdot \frac{(e^x+e^1)(\sin(x))' - \sin(x)(e^x+e^1)'}{(e^x+e^1)^2}$$

$$= \boxed{\frac{2\sin(x)}{e^x+e}\cdot\frac{\cos(x)(e^x+e) - \sin(x)(e^x+0)}{e^x+e}}$$

No reason to simplify

(d) $x = \dfrac{e^y + e^{-y}}{2} = \frac{1}{2}(e^y + e^{-y})$

$$\frac{dx}{dy} = x' =$$

Last time we used quotient rule for $\frac{d}{dy}(e^{-y})$, but with chain rule, it's easier!

$$= \frac{1}{2}\left(\frac{d}{dy}e^y + \frac{d}{dy}e^{-y}\right)$$

$$\overset{CR}{=} \frac{1}{2}\left(e^y + e^{-y}\cdot(-y)'\right) = \boxed{\frac{1}{2}(e^y - e^{-y})}$$

2. If $F(x) = f(xf(xf(x)))$, where $f(1) = 2, f(2) = 3, f'(1) = 4, f'(2) = 5$, and $f'(3) = 6$, find $F'(1)$.

To find $F'(x)$, work "outside-in"

$$F'(x) \overset{CR}{=} f'(xf(xf(x)))\cdot(xf(xf(x)))'$$

$$\overset{PR}{=} f'(xf(xf(x)))\cdot\left(\overset{=1}{x'}\cdot f(xf(x)) + x\,(f(xf(x)))'\right)$$

$$\overset{CR}{=} f'(xf(xf(x)))\cdot\left(f(xf(x)) + xf'(xf(x))\cdot(xf(x))'\right)$$

$$\overset{PR}{=} f'(xf(xf(x)))\cdot\left[f(xf(x)) + xf'(xf(x))\cdot\left(\overset{=1}{x'}f(x) + xf'(x)\right)\right]$$

OK, now plug in & simplify

$$F'(1) = f'(1\cdot f(1\cdot \underbrace{f(1)}_{2}))\cdot\left[f(1\cdot\underbrace{f(1)}_{2}) + 1\cdot f'(1\cdot\underbrace{f(1)}_{2})\cdot\left(\underbrace{f(1)}_{2} + 1\cdot\underbrace{f'(1)}_{4}\right)\right]$$

$$= f'(\underbrace{f(2)}_{3})\cdot\left[\underbrace{f(2)}_{3} + \underbrace{f'(2)}_{5}(2+4)\right]$$

$$= \underbrace{f'(3)}_{6}\cdot(3 + 5\cdot 6)$$

$$= 6\cdot 33 = \boxed{198}$$ whew! done!