MATH 220: CALCULUS I
WORKSHEET 10
FEBRUARY 19, 2013

List of some short-cut rules:

$\frac{d}{dx}(x) = 1$	$\frac{d}{dx}(c) = 0$	$\frac{d}{dx}(x^n) = nx^{n-1}$	$\frac{d}{dx}(e^x) = e^x$
$\frac{d}{dx}(a^x) = a^x \ln(a)$	$\frac{d}{dx}(\ln x) = \frac{1}{x}$	$\frac{d}{dx}(\sin x) = \cos x$	$\frac{d}{dx}(\cos x) = -\sin x$
$\frac{d}{dx}(\tan(x)) = \sec^2(x)$	$\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x)$	$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$	$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(cf(x)) = cf'(x)$ $\quad$ $\frac{d}{dx}(f(x)+g(x)) = f'(x)+g'(x)$

$\frac{d}{dx}(f(x)g(x)) = f'(x)g(x)+f(x)g'(x)$ $\quad$ $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x)-f(x)g'(x)}{(g(x))^2}$

Derivative = slope of the tangent line = instantaneous rate of change

Homework: In Section 3.1, do problems 3-30, 33, 35, 47, 51, 53
In Section 3.2, do the odd problems from 3–33

1. Find an equation of the tangent line to the curve $y = \dfrac{x}{1+x^2}$ at the point $\left(3, \dfrac{3}{10}\right)$.

$$f'(x) = y' = \frac{(1+x^2)(x)' - x(1+x^2)'}{(1+x^2)^2} = \frac{1+x^2 - x(2x)}{(1+x^2)^2} = \frac{1-x^2}{(1+x^2)^2}$$

slope $\rightarrow$ $f'(3) = \dfrac{1-9}{(1+9)^2} = -\dfrac{8}{100} = \boxed{-\dfrac{2}{25}}$

line $\rightarrow$ $y - y_1 = m(x-x_1) = \boxed{y - \frac{3}{10} = -\frac{2}{25}(x-3)}$

2. Use the quotient rule to find the derivatives of $\cot(x)$ and $\csc(x)$.

$$(\cot(x))' = \left(\frac{\cos(x)}{\sin(x)}\right)' = \frac{\sin(x)\cdot(\cos(x))' - \cos(x)(\sin(x))'}{(\sin(x))^2} = \frac{-\sin^2(x) - \cos^2(x)}{\sin^2(x)} = \frac{-(\sin^2(x)+\cos^2(x))}{\sin^2(x)} = \frac{-1}{\sin^2(x)}$$

$$= \boxed{-\csc^2(x)}$$

$$\frac{d}{dx}\csc(x) = \frac{d}{dx}\frac{1}{\sin(x)} = \frac{\sin(x)\cdot(1)' - 1\cdot(\sin x)'}{\sin^2 x} = \frac{0 - \cos x}{\sin^2 x} = \frac{-1}{\sin x}\cdot\frac{\cos x}{\sin x}$$

$$= \boxed{-\csc(x)\cot(x)}$$

3. Differentiate.

(a) $f(x) = \dfrac{x^2\cos(x)}{x - x^2}$

need product rule.

$$f'(x) = \frac{(x-x^2)(x^2\cos(x))' - x^2\cos(x)(x-x^2)'}{(x-x^2)^2}$$

this is an ok final answer also

$$= \boxed{\frac{(x-x^2)(2x\cos(x) + x^2(-\sin(x))) - x^2\cos(x)(1-2x)}{(x-x^2)^2}}$$

$$= \frac{x^2\cos(x) - x^3\sin(x) + x^4\sin(x)}{x^2(1-x)^2} = \frac{\cos(x) - x\sin(x) + x^2\sin(x)}{(1-x)^2}$$

(b) $g(t) = \dfrac{t - t^2}{t^2 - 3t + 2}$

$$g'(t) = \frac{(t^2-3t+2)(t-t^2)' - (t-t^2)(t^2-3t+2)'}{(t^2-3t+2)^2}$$

$$= \frac{(t^2-3t+2)(1-2t) - (t-t^2)(2t-3)}{(t^2-3t+2)^2}$$

OR factor first

$$g(t) = \frac{t\cancel{(1-t)}}{\cancel{(t-1)}(t-2)} = \frac{-t}{t-2}$$

so $$g'(t) = \frac{(t-2)(-t)' - (-t)(t-2)'}{(t-2)^2} = \frac{-(t-2)+t}{(t-2)^2} = \boxed{+\frac{2}{(t-2)^2}}$$ ← much nicer than

(c) $f(x) = \dfrac{e^x - e^{-x}}{2} = \frac{1}{2}\left(e^x - \frac{1}{e^x}\right)$

$$f'(x) = \frac{1}{2}\left((e^x)' - \left(\frac{1}{e^x}\right)'\right) = \frac{1}{2}\left(e^x - \frac{e^x(1)' - 1(e^x)'}{(e^x)^2}\right) = \frac{1}{2}\left(e^x - \frac{0-e^x}{e^{2x}}\right) = \frac{1}{2}\left(e^x + \frac{1}{e^x}\right)$$

quotient rule

$$= \boxed{\frac{e^x + e^{-x}}{2}}$$

(d) $h(w) = \dfrac{w^{2/3}}{e^{w}}$

$$h'(w) = \frac{e^w(w^{2/3})' - w^{2/3}(e^w)'}{(e^w)^2} = \frac{e^w\left(\frac{2}{3}w^{-1/3}\right) - w^{2/3}e^w}{e^{2w}} = \boxed{\frac{\frac{2}{3}w^{-1/3} - w^{2/3}}{e^w}}$$