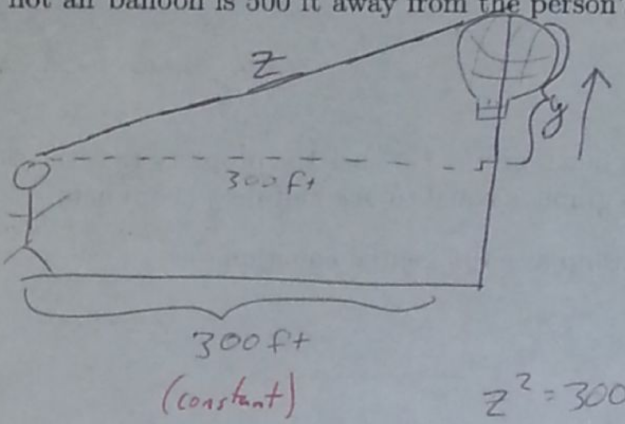
Math 220 (section AD?) **Quiz 6** **Spring 2013**

Name ______________________________

- You have 15 minutes
- No calculators
- Show sufficient work

1. (3 points) A person is standing 300 ft away from the launchpad of a hot air balloon. The hot air balloon lifts off from the ground and travels straight up at a constant rate of 4 ft/sec. How fast is the distance between the hot air balloon and the person's eyes changing when the hot air balloon is 500 ft away from the person's eyes?



Given

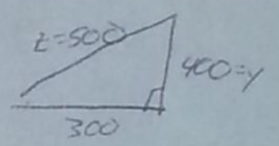
$\frac{dy}{dt} = 4$ ft/sec

want $\frac{dz}{dt}$ when $z = 500$

$z^2 = 300^2 + y^2$ (initial equation)

when $z = 500$, we have $500^2 = 300^2 + y^2$

so y must be 400 ft when $z = 500$ ft



Specific moment in time that we care about.

Take $\frac{d}{dt}$ of initial equation:

$$2z\frac{dz}{dt} = 0 + 2y\frac{dy}{dt}$$

Plug in: $2 \cdot 500 \frac{dz}{dt} = 2 \cdot 400 \cdot 4$

So $\frac{dz}{dt} = \frac{\cancel{2} \cdot 400 \cdot 4}{\cancel{2} \cdot 500} = \boxed{\frac{16}{5} \text{ ft/sec}}$

2. (3 points) A rock is thrown vertically upward from the surface of a planet. The rock's height abve the planet's surface is given by the equation $h(t) = t(24 - 1.2t)$, where t is measured in seconds and $h(t)$ is measured in meters. What is the velocity of the rock when it hits the ground?

Want $h(t)=0$, so set $0 = t(24-1.2t)$ and see that $t=0$ or $\frac{24}{1.2} \to 20$

@ $t=0$, the rock is leaving the ground so we care about $t=20$ sec

To get velocity, take first derivative of $h(t)$

$$h'(t) \overset{PR}{=} 1\cdot(24-1.2t) + t(-1.2) = 24 - 2.4t$$

Since we know the rock hits @ $t=20$, $h'(20) = 24 - 2.4(20) = 24 - 48 = \boxed{-24 \text{ m/s}}$

velocity when rock hits ground

3. (4 points) The graph of a function $y = f(x)$ has the property that the slope of the tangent line at each point on this graph is equal to one half its y-coordinate.

(a) Express this relationship as a differential equation. equation having both a function & a derivative

$(\text{slope}) = \frac{1}{2}(y\text{-val})$, so $\boxed{\frac{dy}{dx} = \frac{1}{2}y}$ (preferred form)

or $f'(x) = \frac{1}{2}f(x)$

or $y' = \frac{1}{2}y$

(b) If the graph goes through the point $(2e^2, 4)$, then find a formula for $f(x)$.

Since y satisfies the diff eqn $\frac{dy}{dx} = ky$ for some k (in this case $k = \frac{1}{2}$)

y is of the form $y = C\cdot e^{kx}$, so $y = C\cdot e^{\frac{1}{2}x}$

Now plug in $x = 2e^2$ & $y = 4$ to find C

$$4 = Ce^{\frac{1}{2}\cdot 2e^2} \to C = \frac{4}{e^{e^2}}$$

Putting it all together, $\boxed{y = \frac{4}{e^{e^2}}\cdot e^{\frac{1}{2}x}}$