Finding Derivatives

1. Find $f'(x)$ given that $f(x) = 4x^{10} + \frac{1}{\sqrt{x}} - \sec x + \ln x$

2. Find $\frac{dv}{dt}$ given that $v = 5t^6 \sin^{-1} (8t)$

3. Find $w'(q)$ given that $w(q) = \frac{\sin (q^3)}{q^4 + 9q}$

4. Find $g'(t)$ given that $g(t) = e^{\cos^2(4t)}$

5. Find $\frac{dy}{dx}$ given that $e^{2y} = x^3y^5 + 6x$

6. Find $g'(t)$ given that $g(t) = 5t^6 - 4t^3 + 10t - e^2$

7. Find $\frac{dv}{dt}$ given that $v = 5t^4 \tan^{-1} t$

8. Find $f'(x)$ given that $f(x) = \frac{\ln x}{x^3 + 4}$

9. Find $h'(t)$ given that $h(t) = \sin (e^{2t})$

10. Find $h'(t)$ given that $h(t) = 40t^3 + \frac{1}{3\sqrt{t}} - 18$

11. Find $\frac{dq}{dt}$ given that $q = 5t^2 \sec t$

12. Find $f'(x)$ given that $f(x) = \frac{x^5}{\ln x}$

13. Find $w'(t)$ given that $w(t) = \tan^{-1} (5t^2)$

14. Find $g'(t)$ given that $g(t) = 5t^5 + \sqrt{t} - 40$

15. Find $f'(x)$ given that $f(x) = \frac{x^5}{\sin x}$

16. Find $P'(t)$ given that $P(t) = (t^9 - 10t^4 + 12)^8$

17. Evaluate the following derivatives.

(a) $\frac{d}{dx} (\cos x) =$

(b) $\frac{d}{dx} (\csc x) =$

(c) $\frac{d}{dx} (\tan x) =$

(d) $\frac{d}{dx} (\sin^{-1} x) =$

(e) $\frac{d}{dx} (\ln x) =$
**Implicit Differentiation**

1. A spherical balloon is inflated at a constant rate of 5 \( ft^3/min \). How quickly is the balloon’s radius increasing at the instant the volume is 20 \( ft^3 \)?

2. A particle moves along the curve \( y = \frac{4}{5}x^2 \). As the particle passes through the point (3,4), its \( x \)-coordinate increases at a rate of 15 cm/s. How fast is the distance from the particle to the origin changing at this instant?

3. Find \( \frac{dy}{dx} \) given that \( \sin(x^2 + y^3) = 5y + 8x \). It is okay to leave your answer in terms of both \( x \) and \( y \).

4. Find the slope of the line tangent to the curve \( x^2y^3 = 3x - 2y \) at the point (2,1).

5. A particle is moving along the curve \( y = \sqrt{1 + x^3} \). As it reaches the point (2,3), the \( y \)-coordinate is increasing at a rate of 18 cm/sec. How fast is the \( x \)-coordinate of the point changing at that instant?

6. Find \( \frac{dy}{dx} \) given that \( x^5e^y = 2x^3 + 5y^2 + 6 \). It is okay to leave your answer in terms of both \( x \) and \( y \).

7. A small balloon is released at a point 40 feet away from an observer, who is on level ground. If the balloon goes straight up at a rate of 10 feet per second, how fast is the distance from the observer to the balloon increasing when the balloon is 30 feet high?

8. A ball is tossed straight up with an initial velocity of 16 feet per second. The ball is 5 feet above the ground when it is released. Its height at time \( t \) is given by \( h = -16t^2 + 16t + 5 \).

What is the ball’s maximum height?

**Exponential Functions**

1. The graph of one of the solutions to the differential equation \( \frac{dy}{dx} = y/2 \) passes through the point (0,6). Determine the \( x \)-value at which this graph intersects the line \( y = 30 \).

2. Determine a formula for \( w \) as a function of \( s \) so that \( \frac{dw}{ds} = 10s \) and \( w(1) = 2 \).

3. Determine a formula for \( w \) as a function of \( s \) so that \( \frac{dw}{ds} = 10w \) and \( w(1) = 2 \).

4. The graph of a function \( y = f(x) \) has a \( y \)-intercept of 8 and has the property that the slope of the curve at every point \( P \) is twice the \( y \)-coordinate of \( P \). What is the equation of the curve?

**Optimization**

1. For the curve \( y = e^{4x} - 3e^{-2x} \), give the \( x \)-value at which the tangent line has the smallest slope.

2. Suppose that a function \( f(x) \) has first derivative given by \( f'(x) = -2e^{x/2}(x^2 - 7x + 14) \). Determine the largest open interval upon which the graph of \( f(x) \) is concave up.

3. A poster is to contain 1000 \( \text{cm}^2 \) of printed matter with margins of 4 cm each at top and bottom and 2 cm at each side. Find the overall dimensions if the total area of the poster is a minimum.
Evaluate the following limits

1. \( \lim_{x \to 1^+} \frac{\sin(5x)}{\ln x} \)
2. \( \lim_{x \to \infty} \frac{\ln x}{x^3} \)
3. \( \lim_{x \to 0^+} \left( \frac{2}{x} - \frac{10}{e^{5x} - 1} \right) \)
4. \( \lim_{x \to 1} \frac{x^2 + 3x - 4}{\sin(x - 1)} \)
5. \( \lim_{x \to \pi/4} \frac{4x - \pi}{4 \tan x} \)
6. \( \lim_{x \to \infty} \left( 1 - \frac{2}{x} \right)^{3x} \)
7. \( \lim_{x \to 0^+} \frac{\ln(x^3 + 3x)}{\ln x} \)
8. \( \lim_{x \to \infty} x^{200} e^{-x} \)
9. \( \lim_{x \to 0} \frac{1 - x - e^{-x}}{x^2} \)
10. \( \lim_{x \to \infty} \frac{\sqrt{x}}{\ln x} \)
11. \( \lim_{x \to \infty} \left( 1 - \frac{1}{2x} \right)^{3x} \)

Graphing, Min/Max

1. A function \( f(x) \) has the following second derivative.
   \( f''(x) = (x + 5)^2 - 4 \)
   What is the largest open interval upon which the graph of \( f(x) \) is concave down?

2. A function \( g(x) \) has the following derivative \( g'(x) = 5e^x(x - 1)^2(x - 2)^3(x - 3)^4 \).
   Determine the \( x \)-value for each local maximum and local minimum on the graph of \( g(x) \).

3. A function \( f(x) \) has first derivative \( f'(x) = e^{0.5x}(10x - 60) \).
   (a) Upon which interval is \( f(x) \) increasing?
   (b) Upon which interval is the graph of \( f(x) \) concave down?

4. Upon which interval is the graph of \( f(x) = 3x^4 - 20x^3 + 10 \) increasing?

5. A function \( f(x) \) has the following second derivative.
   \( f''(x) = 8e^x (x - 6)^2 (2x - 9) (x^2 + 25) \)
   Find the \( x \)-value for each inflection point on the graph of \( f(x) \).

6. Find the coordinates \((x, y)\) for the highest point on the graph of the function \( g(x) = 180x - 10e^{2x} \).