Finding Derivatives

1. Find $f'(x)$ given that $f(x) = 4x^{10} + \dfrac{1}{\sqrt[4]{x}} - \sec x + \ln x$

2. Find $\dfrac{dv}{dt}$ given that $v = 5t^6 \sin^{-1}(8t)$

3. Find $w'(q)$ given that $w(q) = \dfrac{\sin(q^3)}{q^4 + 9q}$

4. Find $g'(t)$ given that $g(t) = e^{\cos^2(4t)}$

5. Find $\dfrac{dy}{dx}$ given that $e^{2y} = x^3y^5 + 6x$

6. Find $g'(t)$ given that $g(t) = 5t^6 - 4t^3 + 10t - e^2$

7. Find $\dfrac{dv}{dt}$ given that $v = 5t^4 \tan^{-1} t$

8. Find $f'(x)$ given that $f(x) = \dfrac{\ln x}{x^3 + 4}$

9. Find $h'(t)$ given that $h(t) = \sin\left(e^{2t}\right)$

10. Find $h'(t)$ given that $h(t) = 40t^3 + \dfrac{1}{3\sqrt{t}} - 18$

11. Find $\dfrac{dq}{dt}$ given that $q = 5t^2 \sec t$

12. Find $f'(x)$ given that $f(x) = \dfrac{x^5}{\ln x}$

13. Find $w'(t)$ given that $w(t) = \tan^{-1}\left(5t^2\right)$

14. Find $g'(t)$ given that $g(t) = 5t^5 + \sqrt{t} - 40$

15. Find $f'(x)$ given that $f(x) = \dfrac{x^5}{\sin x}$

16. Find $P'(t)$ given that $P(t) = \left(t^9 - 10t^4 + 12\right)^8$

17. Evaluate the following derivatives.

    (a) $\dfrac{d}{dx}(\cos x) =$

    (b) $\dfrac{d}{dx}(\csc x) =$

    (c) $\dfrac{d}{dx}(\tan x) =$

    (d) $\dfrac{d}{dx}\left(\sin^{-1} x\right) =$

    (e) $\dfrac{d}{dx}(\ln x) =$

Implicit Differentiation

1. A spherical balloon is inflated at a constant rate of 5 ft^3/min. How quickly is the balloon's radius increasing at the instant the volume is 20 ft^3 ?

2. A particle moves along the curve $y = \frac{4}{9}x^2$. As the particle passes through the point $(3, 4)$, its x-coordinate increases at a rate of 15 cm/s. How fast is the distance from the particle to the origin changing at this instant?

3. Find $\dfrac{dy}{dx}$ given that $\sin\left(x^2 + y^3\right) = 5y + 8x$. It is okay to leave your answer in terms of both x and y.

4. Find the slope of the line tangent to the curve $x^2y^3 = 3x - 2y$ at the point $(2, 1)$.

5. A particle is moving along the curve $y = \sqrt{1 + x^3}$. As it reaches the point $(2, 3)$, the y-coordinate is increasing at a rate of 18 `cm/sec`. How fast is the x-coordinate of the point changing at that instant?

6. Find $\dfrac{dy}{dx}$ given that $x^5e^y = 2x^3 + 5y^2 + 6$. It is okay to leave your answer in terms of both x and y.

7. A small balloon is released at a point 40 feet away from an observer, who is on level ground. If the balloon goes straight up at a rate of 10 feet per second, how fast is the distance from the observer to the balloon increasing when the balloon is 30 feet high?

8. A ball is tossed straight up with an initial velocity of 16 feet per second. The ball is 5 feet above the ground when it is released. Its height at time t is given by $h = -16t^2 + 16t + 5$.

   What is the ball's maximum height?

Exponential Functions

1. The graph of one of the solutions to the differential equation $\dfrac{dy}{dx} = y/2$ passes through the point $(0, 6)$. Determine the x-value at which this graph intersects the line $y = 30$.

2. Determine a formula for w as a function of s so that $\dfrac{dw}{ds} = 10s$ and $w(1) = 2$.

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4. The graph of a function $y = f(x)$ has a y-intercept of 8 and has the property that the slope of the curve at every point P is twice the y-coordinate of P. What is the equation of the curve?

Optimization

1. For the curve $y = e^{4x} - 3e^{-2x}$, give the x-value at which the tangent line has the smallest slope.

2. Suppose that a function $f(x)$ has first derivative given by $f'(x) = -2e^{x/2}\left(x^2 - 7x + 14\right)$. Determine the largest open interval upon which the graph of $f(x)$ is concave up.

3. A poster is to contain 1000 $\texttt{cm}^2$ of printed matter with margins of 4 `cm` each at top and bottom and 2 `cm` at each side. Find the overall dimensions if the total area of the poster is a minimum.

Evaluate the following limits

1. $\lim\limits_{x\to 1^+} \dfrac{\sin(5x)}{\ln x}$

2. $\lim\limits_{x\to\infty} \dfrac{\ln x}{x^3}$

3. $\lim\limits_{x\to 0^+} \left(\dfrac{2}{x} - \dfrac{10}{e^{5x}-1}\right)$

4. $\lim\limits_{x\to 1} \dfrac{x^2+3x-4}{\sin(x-1)}$

5. $\lim\limits_{x\to\pi/4} \dfrac{4x-\pi}{4\tan x}$

6. $\lim\limits_{x\to\infty} \left(1-\dfrac{2}{x}\right)^{3x}$

7. $\lim\limits_{x\to 0^+} \dfrac{\ln(x^3+3x)}{\ln x}$

8. $\lim\limits_{x\to\infty} x^{200}e^{-x}$

9. $\lim\limits_{x\to 0} \dfrac{1-x-e^{-x}}{x^2}$

10. $\lim\limits_{x\to\infty} \dfrac{\sqrt{x}}{\ln x}$

11. $\lim\limits_{x\to\infty} \left(1-\dfrac{1}{2x}\right)^{3x}$

Graphing, Min/Max

1. A function $f(x)$ has the following second derivative.
   $f''(x) = (x+5)^2 - 4$
   What is the largest open interval upon which the graph of $f(x)$ is concave down?

2. A function $g(x)$ has the following derivative $g'(x) = 5e^x(x-1)^2(x-2)^3(x-3)^4$.
   Determine the x-value for each local maximum and local minimum on the graph of $g(x)$.

3. A function $f(x)$ has first derivative $f'(x) = e^{0.5x}(10x-60)$.
   (a) Upon which interval is $f(x)$ increasing?
   (b) Upon which interval is the graph of $f(x)$ concave down?

4. Upon which interval is the graph of $f(x) = 3x^4 - 20x^3 + 10$ increasing?

5. A function $f(x)$ has the following second derivative.
   $f''(x) = 8e^x(x-6)^2(2x-9)\left(x^2+25\right)$
   Find the x-value for each inflection point on the graph of $f(x)$.

6. Find the coordinates (x, y) for the highest point on the graph of the function $g(x) = 180x - 10e^{2x}$.