12 Sept 2014

Math 181

Pigeonhole Principle: Placing more than $k \cdot n$ objects into k classes puts more than n objects into some class.

Example: Given five points in the plane with integer coordinates, the midpoint of the segment joining *some* pair also has integer coordinates.

The midpoint between two points (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$. These coordinates are integers exactly when x_1 and x_2 are both odd or both even, and likewise for y_1 and y_2. The parities of the points can be (odd, even), (even, odd), (even, even), and (odd, odd). With five points, two of them must be in the same class.

Ramsey Theory: *How many elements of some structure must there be to guarantee that a particular property will hold?*

Your home was to answer the question: How many people can be invited to a party so that no group of three people pairwise know each other and no group of three people are pairwise strangers?

If I change "three" to "four", then the answer is 18. Famous mathematician Paul Erdős said a variation of the following:

If a vastly more powerful group of aliens come to earth and threaten to destroy our planet unless we determine the value for "five", then we should combine all the computing power in the world to try to find the value. If instead they ask for the value for "six"... we're better off trying to fight the aliens.

Definitions: The *complete graph* on n vertices is denoted by K_n.

The *Ramsey number* is denoted $R(p, q)$ and is the smallest number of people that can be invited to a party such that you are *guaranteed* to have either p people that pairwise know each other or q people that are pairwise strangers.

In the graph theory context, $R(p, q)$ is the minimum number of vertices in a complete graph such that *any* red–blue coloring of the edges admits a red K_p or a blue K_q.

Reminder: The first midterm exam is in two weeks (Sept 26).