10 Sept 2014
Math 181

Generalized Colorings: Consider the FCC radio station example from last time. There are still more questions we can ask. First, interference is not a simple cutoff that can be established at 500 miles. Secondly, merely having different frequencies is not sufficient to avoid interference.

Suppose we have the following rule: If two towers are within 600 miles of each other, their frequencies must be different, but if they are within 300 miles of each other, their frequencies must differ by at least 2.

How would you model this problem using a graph? (This problem is known as $L(2,1)$-labelling, and has been used in industry to solve problems with interference.)

	A	B	C	D	E	F	G
A	-	450	550	700	600	850	900
B	450	-	500	300	250	600	750
C	550	500	-	100	530	800	900
D	700	300	100	-	470	650	700
E	600	250	530	470	-	350	490
F	850	600	800	650	350	-	530
G	900	750	900	700	490	530	-

Here's another question: Some time after I solve the problem of assigning frequencies, a new station would like to build a tower that would be in range of some of the existing towers, causing interference. Where should I tell the station to build the tower so that I can assign a frequency without increasing the amount of spectrum being used?

A Game: When we got married, my wife and I moved into a split-level house (not really, but just go with it). Drawing a vertex for each room or hallway, the house looks like this:
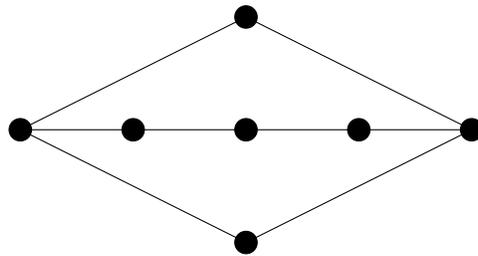
All of the rooms were white when we moved in, and so my wife wanted (me) to paint the rooms. Because she had to work, each day she would get some color of paint and give me a list of rooms to paint. If I could simply follow directions, everything would be fine, but I'm a graph theorist! So I cannot allow myself to paint adjacent rooms with the same color. My wife is forgiving, so I can get away not coloring *all* of the rooms she asks for on a particular day. But if she has to ask me more than twice to color a room, then I'm in "trouble".

Can I avoid getting in trouble no matter how she asks me to paint the rooms?

Homework: As a precursor to what we'll discuss on Friday, your homework is to answer the following question:

How many people can I invite to a party so that no group of three people pairwise know each other AND no group of three people are pairwise strangers?