5 Sept 2014

Math 181

Discussion: We are now starting Section 3.5 on Page 93. Your homework was to read the Spotlights on pages 96–97.

Up to this point, edges in a graph have represented paths or distances. We imagined "traveling" along an edge while doing some task (delivering mail, checking meters, etc.). Edges can represent other relationships, however. One of the first examples of a graph we discussed was a "social graph" (i.e. Facebook, Twitter, etc.) where edges in that case represent that two people are friends on some social network.

As we explore other applications of graphs, we now consider the case where edgdes represent "conflicts". Today, we'll cover two examples: Spotlight 3.3 and Spotlight 3.4.

Spotlight 3.3:

1. What is the Four Color Conjecture (now Theorem)?
2. When was the Four Color Conjecture first solved? Who solved it, and where?
3. Instead of coloring vertices, we are coloring faces. These problems are equivalent by an operation known as the *dual*.
4. How would you go about coloring a tree? What is its chromatic number?
5. Trying to determine whether or not a graph can be properly colored with only four colors is NP-complete. But if we only consider plane graphs, then we immediately know that the answer is "yes". A big part of mathematics is taking things that are hard to compute in general and proving theorems that solve the problem for special subcases of the general problem.

Spotlight 3.4:

1. We want to turn this word problem into a graph problem.
   (a) What should we choose as vertices?
   (b) When will we have an edge?
   (c) What does a solution to the problem look like?
2. How would you go about trying to color this graph?

We will spend next week discussing graph coloring in more detail. Most of the material will not be found in the book, so you will need to carefully study the definitions in my notes.

Definitions:

- If two vertices are connected by an edge, we say they are *adjacent*.
- A *vertex coloring* in a graph assigns a color (label) to each vertex in the graph.
- We say a vertex coloring is *proper* (or "is a proper coloring") if adjacent vertices receive distinct colors.
- The *chromatic number* is the minimum number of colors needed to properly color a graph.
- A *plane graph* is a graph that may be drawn in the plane without edges crossing each other.
- In a plane graph, the enclosed regions formed by vertices and edges are called *faces*.
- The *unbounded face* is the face that extends out "to infinity" (or to the outer edge of whatever you are drawing the graph on).
- We obtain the *dual* of a plane graph by placing a vertex inside of each face (including the unbounded face) and drawing an edge between two vertices if the corresponding faces share an edge.
- A *tree* is a connected graph that contains no cycles.
- The *Four Color Theorem* states that the chromatic number of any plane graph is at most 4; stated another way, it says that the regions of any "map" can be properly colored by using at most four colors.

Homework:

Study the definitions. Wednesday's quiz will focus on stating and using definitions.