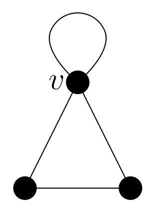
29 Aug 2014

Math 181

Housekeeping: The topics in this course are inherently visual. I will put many pictures and examples into my notes and handouts, but there will still be things that I put up on the board. While I encourage good note-taking, sometimes it is difficult to copy all of the information from a picture on the board. For this reason, **taking pictures of the board is allowed** as long as it is not disruptive.

Loops:



When asked on Wednesday's quiz, a majority of students stated that adding a loop at a vertex should only increase the valence by 1. This definition is certainly valid when the valence is defined by the number of edges containing a particular vertex.

To take the opposing view, I offer an example where a loop should increase the valence by 2.

Question: Does the graph above have an Euler circuit?

The Euler Circuit Theorem states that if a graph has a vertex with odd valence, then it is impossible to have an Euler circuit. If we count the loop as increasing valence by 1, then v has valence 3. But there is a natural Euler circuit for this graph.

In fact, I can add loops (possibly many) to **any** vertex and not change whether or not it has an Euler circuit. Neither definition for how valence counts loops is wrong, but in mathematics, we must be careful about the consequences of certain definitions. The "loops count as 1" definition for valence means thta the Euler Circuit Theorem is false unless we forbit these loops.

Traveling Salesman: It is not always best to try to visit all edges exactly once. Sometimes, the goal is only to visit all vertices exactly once. Page 44 in the book has a great example: We consider a list of cities, and we are given the distances between all pairs of cities. Our goal is to visit all of the cities exactly once while traveling the least distance overall.

Definitions:

- A *digraph* is a graph in which every edge is directed (has an arrow indicating a direction of travel).
- We may then talk about the *out-valence* or *in-valence* of a vertex.
- Given a list of cities, and "costs" or "distances" between all pairs of cities, the *traveling salesman problem (TSP)* involves finding the trip of minimum cost that a salesman can make to visit all the cities.