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Math 181

Last time, we discussed what graphs can be used to model (for example, social networks and computer networks). Today, we will talk about how graphs can be used to solve problems. The phrase "time is money" is not always accurate; To a business, however, any process that takes a long time to complete is a drain on resources, and it would be beneficial to the business to complete the process in an optimal way.

To solve real-life problems using mathematics, we must first find an appropriate mathematical model of the problem. It should have the property that a solution for the model is a solution to real-life problem.

Application 1: Parking Meters
Suppose we have a map of roads, and on that map, some blocks have parking meters. Those parking meters must be checked by an officer on foot. The goal is to check all of the parking meters in the least amount of time. We must then ask what a *solution* to this problem looks like, and the answer is "a particular route that the officer takes to check all meters".

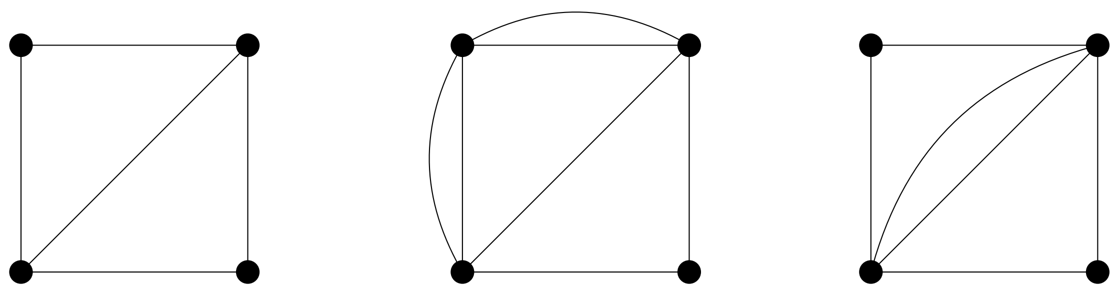
One way we can do this is by making a graph. At each intersection, we create a vertex, and any time a side of the street has parking meters, we add an edge between the corresponding intersections.

By finding an Euler circuit in the graph, we simultaneously find a route for the officer that checks all meters!

Application 2: Chinese Postman Problem
This variant of the parking meter problem is very similar. We would *like* to find an Euler circuit, but we know that in practice, one may not exist. So we instead seek to retrace our steps as little as possible.

The problem gets its name from the Chinese mathematician Meigu Guan in 1962, and it involves a postman delivering mail to every house (instead of an officer checking meters).

The blocks that we must retrace, we add into our graph so that our final route is an Euler circuit for *some* graph (but maybe not the original graph). This process is called *eulerization*, and some are better than others.



The first graph does not contain an Euler circuit because it has two vertices of odd valence.

The second graph does have an Euler circuit, so it is an eulerization of the first graph.

The third graph is a *better* eulerization because it involves retracing yours steps on only one edge instead of two.

Definitions:

- A graph is *connected* if there exists a path between any pair of vertices.
- If a graph is not connected, then it has multiple *components*.
- An *Euler circuit* in a graph is a circuit that visits each edge exactly once.
- An *eulerization* of a connected graph is obtained by duplicating existing edges in the graph until all vertices have even valence. This process is called *eulerizing* a graph.