3 Oct 2014

Math 181

Mixed Strategies

The scene you watched is from *A Princess Bride*. Westley tricks Vizzini into drinking poison, but not before Vizzini went into a long game-theoretical deduction.

There are two payoff matrices in this case: There's the game Vizzini *thinks* he's playing, and there's the game thta Westley sets up.

		Vizzini	
		Near Cup	Far Cup
Westley	Near Cup		
	Far Cup		

Vizzini's Game

	Vizzini	
	Near Cup	Far Cup
Near Cup		
Far Cup		

Actual Game

Vizzini illustrates two mistakes of game theory. The first is that the techniques in game theory only make sense when *both* players have total knowledge of the game being played. The second is that trying to figure out what your opponent will do is not the best solution to the game.

Rather than try to determine exactly what the other player will do, we can instead employ a mixed strategy (defined below) that will account for all possible outcomes.

Let's return to the example from last time:

		Lisa	
		Hwy 1	Hwy 3
Henry	Route A	10	6
	Route C	2	7

1. Compute the maximin/minimax values for this game. (Review from last time)
2. There's no saddlepoint, but what can you say about the *value* of this game?
3. Suppose Lisa choses Highway 1 with probability p and Highway 3 with probability $1-p$. What is the expected value for Lisa if Henry chooses A? What about if Henry chooses C? Denote these values by E_A and E_C.
4. What value of p will guarantee the best outcome for Lisa?
5. Suppose Henry chooses A with probability q and C with probability $1-q$. Determine the expected values E_1 and E_3 for Henry when Lisa choses Highway 1 or 3, respectively.
6. What value of q will guarantee the best outcome for Henry?
7. Determine the value of this game.

Definitions:

A *pure strategy* is one where the player chooses the same fixed action every time.

A *mixed strategy* is one where the player assigns fixed probabilities to each of the pure strategies to govern the final choice to be made.

If there are n outcomes with payoffs $s_1, s_2, \ldots, s_n$ that occur with probabilities $p_1, p_2, \ldots, p_n$, then the average, or *expected value*, is $p_1 s_1 + p_2 s_2 + \cdots + p_n s_n$.

A *fair game* is one that has value 0 (favors neither player). If a player uses an optimal strategy he or she will not lose.

Homework: Read 15.3, pages 543–549. Go to `www.tinyurl.com/gt101mahoney`. Watch #5 (What is a Nash Equilibrium?) and #7 (Mixed Strategy Nash Equilibrium and Matching Pennies).