

19 Sept 2014

Math 181

Ramsey Number Definitions: Here I will describe $R(p, q)$ in several ways, all of which are equivalent.

- 1) The *Ramsey number* $R(p, q)$ is the smallest number of vertices in a red–blue edge-colored complete graph that must have (cannot avoid) either a red K_p or a blue K_q .
- 2) This also means that $R(p, q) - 1$ is the largest number of vertices in a complete graph that has a red–blue coloring of the edges that has no red K_p and no blue K_q .
- 3) $R(p, q) = k$ means two things: first, that there is a red–blue coloring of the edges of K_{k-1} that avoids a red K_p and a blue K_q , and second that any red–blue coloring of the edges of K_k must have a red K_p or a blue K_q .

Pigeonhole Principle: The Pigeonhole Principle can answer some surprising questions. Let's take a look at one such example.

Lemma: This is a useful trick that you should know:

If a and b have the same remainder when divided by N , then $a - b$ is a multiple of N .

Practice Problems:

1. Eight positive numbers are chosen at random. Explain why two of them are sure to differ by a multiple of seven.
2. Let $x_1, x_2, x_3, \dots, x_{20}$ be 20 *consecutive* integers. Show that if any 11 of them are chosen at random, then there will be some pair of chosen integers that differ by 10.
3. Fifty-one integers are chosen at random from the numbers 1 to 100. Prove that at least two of the chosen integers will differ by 10.

Homework: To choose the future topics that we'll cover in the course, I want your feedback! For Wednesday, I want each of you to find three topics that you find interesting and would like to learn more about.

One topic must be from the book—a subsection, a spotlight, or even a whole chapter. One topic must come from elsewhere; this could be of the form “where is the math in [topic]?”, it can be a particular problem that you would like to solve or know more about, or it can be something you find online that interests you. The third topic can be something else from

the book, or it can be from online.

If you need help finding such topics, send me an email. I know of many online resources that have fun or interesting math-related topics.

Midterm: Friday, Sept 26. On Monday, Sept 22, I'll give a review guide. If you want to start studying earlier, any definition up to this point will certainly be fair game for the exam.