17 Sept 2014

Math 181

Things to remember:

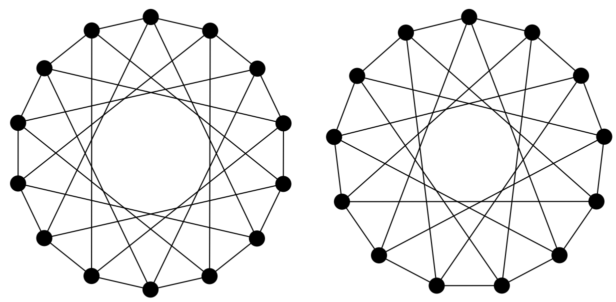
The *Pigeonhole Principle* states that if I have at least $kn + 1$ objects that I'm placing into k categories, then some category will have at least $n + 1$ objects in it.

The *Ramsey number* $R(p, q)$ is the smallest number of vertices in a red–blue edge-colored complete graph that must have either a red K_p or a blue K_q.

Ramsey's Theorem: $R(p, q) \leq R(p - 1, q) + R(p, q - 1)$. (Actually, this is only a special case of Ramsey's Theorem, but it's the only one we'll use)

Midterm: Friday, Sept 26.

Fixing the Homework: I wanted you to show that $R(3, 5) = 14$, but I messed up.



Wrong! (14 vertices) Correct! (13 vertices)

Recall (i.e. midterm's coming up, don't forget) that to show that the chromatic number of a graph equals some number k, you must do two things:

1. Give a proper coloring using k colors.
2. Show that it is not possible to give a proper coloring with $k - 1$ colors.

The first step shows that the chromatic number is *at most* k, and the second step shows that the chromatic number is *greater than* $k - 1$.

Similarly, to show that $R(p, q) = n$, you must also do two things:

1. Give a red–blue coloring of K_{n-1} that has no red K_p *and* no blue K_q.
2. Show that every red–blue coloring of K_n *must* have either a red K_p *or* a blue K_q.

The inequalities are reversed from the chromatic number case. In the first step, we are showing that $R(p, q) > n - 1$, and in the second step, we show that $R(p, q) \leq n$. Typically, it is the second step that proves to be much more difficult.

In the above example (using the graph on the right), we think about coloring all of the visible edges as being red, and thus any edge that is *not* present in the above graph, we think of as being blue.

Why do we do this? Because the edges in the graph do not form any triangles! So if the blue edges do not form any K_5, then this coloring shows that $R(3, 5) > 13$. In the graph, since the blue edges are the ones missing, it will be easier to show that every group of 5 vertices will contain *some* red edge (and thus, there is no all-blue K_5).