

19 Nov 2014

Math 181

### Practice Problems

1. What is the probability of rolling exactly one six when a pair of 6-sided dice is rolled?
2. Roll a pair of 6-sided dice. Let  $A$  be the event that the sum is 2, and let  $B$  be the event that the sum is 12. Does it follow that  $P(A \text{ or } B) = P(A) + P(B)$ ?
3. Suppose you are dealt two cards from a standard 52-card deck. What is the probability that both cards are the same suit?
4. Every number has a unique prime factorization. Using only the first five prime numbers (2,3,5,7,11), how many numbers with exactly 4 factors (some primes may be repeated, for example  $36 = 2^2 3^2$  meets the criteria) can be created?  
Some examples:  $36 = 2^2 \cdot 3^2$ ,  $210 = 2 \cdot 3 \cdot 5 \cdot 7$ , and  $2662 = 11^3 \cdot 2$  all meet the criteria, but  $70 = 2 \cdot 5 \cdot 7$  does not (only 3 factors),  $104 = 2^3 \cdot 13$  does not (has a prime larger than 11), and  $32 = 2^5$  does not (too many factors).
5. Flip a coin. Player A gets a point if it is heads, and Player B gets a point if it is tails. The game ends when a player reaches 3 points. Suppose that the final score is 3 for A and 2 for B. How many sequences of flips produce this outcome?
6. Determine each of the following values:  ${}_n P_0$ ,  ${}_n P_n$ ,  ${}_n C_0$ ,  ${}_n C_n$ .
7. Explain the formula  ${}_n P_k = \frac{n!}{(n-k)!}$ .
8. A note on Benford's law (from Wikipedia):
  - The 1974 Vancouver telephone book violates Benford's law because regulations prevent any number from starting with a "1".
  - The 1960 and 1970 U.S. Census data for populations violates the law because only places with a population of at least 2500 are counted. This truncation causes 1 to begin numbers 19% of the time, and 2 to begin numbers 20% of the time.
  - More examples on Wikipedia: <http://tinyurl.com/tmahone2-benford>
9. Precisely state Arrow's Impossibility Theorem.
10. Can you precisely state each of the four properties (CWC, Pareto, IIA, Monotone)?
11. In a few short sentences, explain why the Borda count satisfies the Pareto condition.