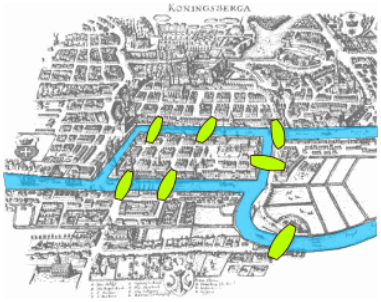
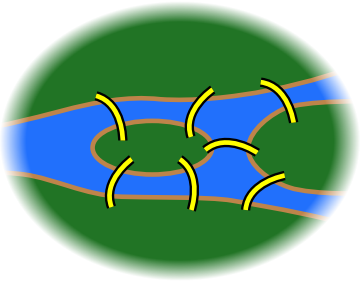
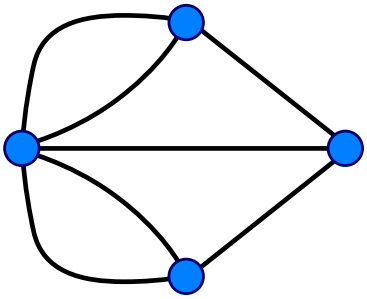
27 Aug 2014

Math 181

Why "Euler" Circuit?

In Euler's time, the German town of Königsberg had seven bridges, shown on the map below. It was asked (as a recreational math problem) whether or not it was possible to walk a route that crosses each bridge exactly once.



Euler showed that it is impossible by using a graph. This is the earliest example of a graph being used to solve a problem in mathematics. Not only did Euler show that there is no *Euler circuit* in this graph, but he proved exactly when a graph does or does not have such a circuit.

Theorem 1 *Euler Circuit Theorem*
If a graph is connected and all vertices have even valence, then the graph has an Euler circuit. Conversely, if any vertex has odd valence, then the graph does *not* have an Euler circuit.

This theorem states that if all valences are even, then there is an Euler circuit. But **how** can you find such a circuit? This theorem does not answer that question. To see an *algorithm* of how to find an Euler circuit (when you know that one exists), see page 13 in the textbook.

In the parking meter problem, if the graph does not have an Euler circuit, how can the problem be solved? Certainly, the answer is not to give up trying to check the meters just because an Euler circuit doesn't exist! Think about what is necessary for a graph to have an Euler circuit, and try to appropriately *modify* the graph to find a "good enough" solution.