Thursday, June 18  **  Projections, Cross Product, and its Applications.

1. Let \( \mathbf{a} = (1, 1) \) and \( \mathbf{b} = (2, -1) \).

   (a) Calculate \( \text{proj}_b \mathbf{a} \) and draw a picture of it together with \( \mathbf{a} \) and \( \mathbf{b} \).

   (b) The orthogonal complement of the vector \( \mathbf{a} \) with respect to \( \mathbf{b} \) is defined by

   \[
   \text{orth}_b \mathbf{a} = \mathbf{a} - \text{proj}_b \mathbf{a}.
   \]

   Calculate \( \text{orth}_b \mathbf{a} \) and draw two copies of it in your picture from part (a), one based at \( 0 \) and the other at \( \text{proj}_b \mathbf{a} \).

   (c) Check that \( \text{orth}_b \mathbf{a} \) calculated in (b) is orthogonal to \( \text{proj}_b \mathbf{a} \) calculated in (a).

   (d) Find the distance of the point \( (1, 1) \) from the line \((x, y) = t(2, -1)\). Hint: relate this to your picture.

2. State whether each expression is meaningful. If not, explain why. If so, state whether it is a vector or a scalar.

   (a) \( \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) \)

   (b) \( \mathbf{a} \times (\mathbf{b} \cdot \mathbf{c}) \)

   (c) \( \mathbf{a} \times (\mathbf{b} \times \mathbf{c}) \)

   (d) \( \mathbf{a} \cdot (\mathbf{b} \cdot \mathbf{c}) \)

   (e) \( (\mathbf{a} \cdot \mathbf{b}) \times (\mathbf{c} \cdot \mathbf{d}) \)

   (f) \( (\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d}) \)

3. A vector is called a unit vector if it has magnitude 1. Find two unit vectors orthogonal to both \((1, 3, 2)\) and \((0, -1, 1)\).

4. Find the area of the parallelogram with vertices \( A(-2, 1), B(0, 4), C(4, 2), \) and \( D(2, -1) \).

5. Find the volume of the parallelepiped determined by the vectors \( \mathbf{a} = (1, 2, 3), \mathbf{b} = (-1, 1, 2), \) and \( \mathbf{c} = (2, 1, 4) \).
6. Use the triple product (mixed product) to determine whether the points \(A(1, 3, 2), B(3, -1, 6), C(5, 2, 0),\) and \(D(3, 6, -4)\) lie in the same plane.

7. (a) Let \(P\) be a point not on the line \(L\) that passes through the points \(Q\) and \(R\). Show that the distance \(d\) from the point \(P\) to the line \(L\) is

\[d = \frac{|a \times b|}{|a|}\]

where \(a = \overrightarrow{QR}\) and \(b = \overrightarrow{QP}\).

(b) Use the formula in part (a) to find the distance from the point \(P(1, 1, 1)\) to the line through \(Q(0, 6, 8)\) and \(R(-1, 4, 7)\).

8. (a) Let \(P\) be a point not on the plane that passes through the points \(Q, R,\) and \(S\). Show that the distance \(d\) from the point \(P\) to the plane is

\[d = \frac{|a \cdot (b \times c)|}{|a \times b|}\]

where \(a = \overrightarrow{QR},\) \(b = \overrightarrow{QS},\) and \(c = \overrightarrow{QP}\).

(b) Use the formula in part (a) to find the distance from the point \(P(2, 1, 4)\) to the plane through the points \(Q(1, 0, 0), R(0, 2, 0),\) and \(S(0, 0, 3)\).