Monday, June 15  

**A review of some important calculus topics**

1. Chain Rule:
   
   (a) Let $h(t) = \sin(\cos(\tan t))$. Find the derivative with respect to $t$.
   
   (b) Let $s(x) = \sqrt[3]{x}$ where $x(t) = \ln(f(t))$ and $f(t)$ is a differentiable function. Find $\frac{ds}{dt}$.

2. Parameterized curves:
   
   (a) Describe and sketch the curve given parametrically by
   
   $$
   \begin{align*}
   x &= 5 \sin(3t) \\
   y &= 3 \cos(3t)
   \end{align*}
   $$

   for $0 \leq t < \frac{2\pi}{3}$.

   What happens if we instead allow $t$ to vary between 0 and $2\pi$?

   (b) Set up, but do not evaluate an integral that calculates the arc length of the curve described in part (a).

   (c) Consider the equation $x^2 + y^2 = 16$. Graph the set of solutions of this equation in $\mathbb{R}^2$ and find a parameterization that traverses the curve once counterclockwise.

3. 1st and 2nd Derivative Tests:
   
   (a) Use the 2nd Derivative Test to classify the critical numbers of the function $f(x) = x^4 - 8x^2 + 10$.
   
   (b) Use the 1st Derivative Test and find the extrema of $h(s) = s^4 + 4s^3 - 1$.

   (c) Explain why the 2nd Derivative test is unable to classify all the critical numbers of $h(s) = s^4 + 4s^3 - 1$.

4. Consider the function $f(x) = x^2 e^{-x}$.
   
   (a) Find the best linear approximation to $f$ at $x = 0$.
   
   (b) Compute the second-order Taylor polynomial at $x = 0$.

   (c) Explain how the second-order Taylor polynomial at $x = 0$ demonstrates that $f$ must have a local minimum at $x = 0$.

5. Consider the integral $\int_{0}^{\sqrt{3}\pi} 2x \cos(x^2) \, dx$.
   
   (a) Sketch the area in the $xy$-plane that is implicitly defined by this integral.

   (b) To evaluate, you will need to perform a substitution. Choose a proper $u = f(x)$ and rewrite the integral in terms of $u$. Sketch the area in the $uv$-plane that is implicitly defined by this integral.

   (c) Evaluate the integral $\int_{0}^{\sqrt{3}\pi} 2x \cos(x^2) \, dx$. 