1. (a) Yearly income: $24,000 (your exact number will vary)

(b) $24,000/12 = 2,000

(c) $m = 12, \ r = .07, \ so, \ l = \frac{.07}{12} \approx .005833\ldots$

(d) This is like purchasing an annuity that has a rate of .07 and makes monthly payments of 2,000 for 30 years.

Formula for finding the principal needed for an annuity:

\[
p = \frac{d}{i} \left[ 1 - (1 + i)^{-n} \right]
\]

\[d = 2000, \ i = .005833\ldots, \ n = 360\]

\[p = \frac{2000}{.005833\ldots} \left[ 1 - (1 + .005833\ldots)^{-360} \right]
\]

\[\approx 309615.14\]

Part (d) tells us how much we need to save up by 65 in order to retire in this situation. Parts (e) and (f) deal with the deposits we need to make in order to have the amount from part (d) by the time we are 65, given two different time frames.
(e) Start saving at 35 \( (s_0, t=30) \)

(i) Making monthly deposits/payments into a retirement account, so we use the savings formula for regular payments, and solve the equation for \( d \) instead of \( A \). \( n = 30 \times 12 = 360 \)

\[
d = A \times \frac{i}{(1+i)^n-1} = \frac{300,615.14 \times 0.005833 \ldots}{(1+0.005833)^{360}-1} \approx 246.41
\]

(ii) We will make \( n \) payments of \( d \) so, total deposited = \( 360 \times 246.41 \approx 88,708.22 \)

(iii) At the end of 30 years, we will have \( 300,615.14 \) in the account. We only deposited \( 88,708.22 \), so the rest is from interest.

\[
I = A - n \times d = 300,615.14 - 88,708.22 = 211,906.92
\]

(F) Same process as (e), but \( t = 40 \), so, \( n = 480 \).

(i) \( d = \frac{300,615.14 \times 0.005833 \ldots}{(1+0.005833)^{480}-1} \times 114.53 \)

(ii) total deposited = \( 480 \times 114.53 \approx 54,973.53 \)

(iii) \( I = 300,615.14 - 54,973.53 = 245,641.61 \)

(g) It is better to start saving at 25 because we will make smaller monthly payments, and in total we don't have to save as much money. We earn more money in interest by starting at 25 than at 35.
2. Student loan with \( P = 6,500 \)
\[ r = 0.043, \quad m = 12 \]

(i) No payments while in school - 2 years

(ii) For two years the loan has no payments and interest is compounded monthly,

So, we use the compound interest formula,

\[ i = \frac{0.043}{12} \approx 0.0035833; \quad n = 24 \]

\[ A = P (1 + i)^n = 6,500 \times (1 + 0.0035833)^{24} \approx 7,082.65 \]

(iii) After graduating, we owe 7082.65. So, this is the amount we need to make payments on, not just 6,500. Let \( P = 7082.65 \). Use the loan amortization formula.

\[ i = 0.0035833, \quad n = 10 \times 12 = 120, \quad t = 10 \]

\[ d = \frac{P}{n} \left[ \frac{i}{1 - (1+i)^{-n}} \right] = \frac{7082.65}{120} \left[ \frac{0.0035833}{1 - (1 + 0.0035833)^{-120}} \right] \approx 72.72 \]

We make 120 payments of 72.72.

So, the total paid on the loan is 8726.71

(iv) Same as (iii) except now \( t = 6 \).

\[ i = 0.0035833, \quad n = 72 \]

\[ d = \frac{7082.65}{72} \left[ \frac{0.0035833}{1 - (1 + 0.0035833)^{-72}} \right] \approx 111.78 \]

Total paid = 72 \times 111.78 = 8048.17
b) Each month, pay the interest charged.

(i) the periodic interest rate is \( i = 0.003583 \).

The interest charged on 6,500 after one month is

\[
P \times i = 6,500 \times 0.003583 \\
= 23.29.
\]

(ii) Because we paid the interest each month, when we graduate we only owe 6,500.

Use the amortization formula,

\[
d = \frac{6,500 \times 0.003583}{1 - (1 + 0.003583)^{-120}} = 66.74
\]

We make 24 payments of 23.29 before graduation and 120 payments of 66.74 after graduation.

Total paid = 24 \times 23.29 + 120 \times 66.74 \approx 567.77

(iii) Same as (ii) but \( t = 6 \).

\[
d = \frac{6,500 \times 0.003583}{1 - (1 + 0.003583)^{-6}} \approx 102.58
\]

Total paid = 24 \times 23.29 + 72 \times 102.58 = 7,945.05

(c) By paying off the interest each month in school, we save almost $160 in the 10 year case. In the 6 year case, we save $103.

By paying interest in school and then paying off the loan, we save $781.66 compared to making no interest payments before graduation and then taking 10 years to pay off the loan.