Math 241, Spring 2007, Merit Worksheet 7

1. Convert the equation into both cylindrical and spherical coordinates:
   (a) $x^2 + y^2 = 2x$
   (b) $z = x^2 - y^2$

2. Describe the graph of the equation:
   (a) $\rho = 4 \cos \phi$.
   (b) $\rho^3 - 4\rho = 0$.

3. Write an equation for the surface generated by revolving this curve around the indicated axis. Then sketch the surface:
   (a) The line $z = 3x$; the $z$-axis.
   (b) $x = 2y^2$; the $x$-axis.

4. Find the domains of the following functions (on $\mathbb{R}^3$):
   (a) $f(x, y, z) = \sqrt{x - y}$
   (b) $f(x, y, z) = \sqrt{1 - x^2 - y^2 - z^2}$
   (c) $f(x, y, z) = \frac{\log xyz}{xy^2 - xy}$
   (d) $f(x, y, z) = 4x^2y^4z^8 + z^2 + \sqrt{1 + x^2}$

   Find where the function in (a) has value 4. Find where the function in (b) has value 0. Find where the function in (d) has value $-1$.

5. The diagram below shows the level curves of a function.

What path will result in the greatest change in altitude? Which path is the steepest?

(a) A to B
(b) A to C
(c) A to D
(d) All the same.
6. Can you think of two or more surfaces which have the following as their level curves?

7. Sketch some typical level curves of the function $f(x, y) = y - x^2$.

8. What are the cylindrical coordinates of a sphere centred at $(0, 0, 2)$ of radius 3?

9. The angle between the vectors $-x\vec{i} - \vec{j} + \vec{k}$ and $x\vec{i} + 2\vec{j} - 3\vec{k}$:
   
   (a) is between 0 and 45 degrees
   (b) is between 45 and 90 degrees
   (c) is greater than 90 degrees
   (d) can be any of the above depending on the value of $x$.

10. Two vectors have a dot product of 14. To guarantee the dot product is equal to 28, you could:
    
    (a) double the angle between the vectors
    (b) double the length of both vectors
    (c) double the length of one vector
    (d) none of the above

**Warm-Up Problems for Next Time**

1. I’ll hold a review session/practice exam on Saturday at 3pm. It will be in the Merit Room.

2. Take the exam Monday at 9. Best of luck!