Math 241, Spring 2007, Merit Worksheet 16

1. Evaluate \( \int_0^1 \int_0^1 e^{\text{max}(x^2, y^2)} \, dx \, dy \), where \( \text{max}(x^2, y^2) \) means the larger of the two numbers \( x^2 \) and \( y^2 \).

2. Switch the order of integration:
\[
\int_{-1}^1 \int_{(x+1)^2}^{2x+2} f(x, y) \, dy \, dx.
\]

3. Use polar coordinates to combine the sum
\[
\int_{1/\sqrt{2}}^1 \int_{\sqrt{1-x^2}}^x xy \, dy \, dx + \int_{1}^\sqrt{3} \int_{0}^x xy \, dy \, dx + \int_{\sqrt{2}}^2 \int_{0}^{\sqrt{4-x^2}} xy \, dy \, dx
\]
into one double integral and then evaluate.

4. Convert to polar coordinates and evaluate:
\[
\int_{-2}^0 \int_{0}^{\sqrt{4-x^2}} (x^2 + y^2) \, dy \, dx.
\]

5. Express the following integral in cartesian coordinates:
\[
\int_{0}^{\pi} \int_{0}^{\pi/2} r \, d\theta \, dr.
\]

6. Evaluate the following integral:
\[
\int \int_R \sqrt{x^2 + y^2} \, dA,
\]
where \( R \) is the region \( R = \{(x, y) \mid 1 \leq x^2 + y^2 \leq 9, \ y \geq 0\} \).

7. The Archimedean spiral is the equation \( r = \theta \). What area is enclosed by the spiral when \( \theta \) ranges from 0 to \( 3\pi \)?

8. Find the volume of the region bounded by the paraboloids \( z = x^2 + 2y^2 \) and \( z = 12 - 2x^2 - y^2 \).

9. Find the volume of the “ice-cream cone” bounded by the sphere \( x^2 + y^2 + z^2 = 4 \) and the cone \( z = \sqrt{x^2 + y^2} \).
10. Evaluate
\[ \int_0^\infty \int_0^\infty \frac{1}{(1 + x^2 + y^2)^2} \, dx \, dy. \]

11. Suppose that \( D \) is the disk with \( x^2 + y^2 \leq 1 \). Find (without integrating)
(a) \( \int \int_D \sqrt{1 - x^2 - y^2} \, dA \)
(b) \( \int \int_D 5 - x^2 \sin x + y^2 \cos y \, dA \)
(c) \( \int \int_D (1 - \sqrt{x^2 + y^2}) \, dA. \)

12. Suppose that \( R \) is the region in the \( xy \)-plane between the two curves \( y = 2x^2 \) and \( y = 2x - 4 \). Find the volume of the region between the surface \( z = x \) and the region \( R \).

**Warm-Up for next time**

1. Find the volume of the solid that lies below the surface \( z = x^2 \) and above the region in the \( xy \)-plane bounded by the curves \( y = x^2 \) and \( y = 1 \).