Math 241, Spring 2007, Merit Practice Exam 2

1. Let \( f(x, y, z) = \sqrt{xyz^3} \) and let \( P \) be the point \((2, 2, 2)\).

   (a) Find the maximum directional derivative of \( f \) at \( P \) and the direction in which it occurs.

   (b) Find the directional derivative of \( f \) at \( P \) in the direction of \( \vec{v} = 3\hat{i} + 12\hat{j} + 4\hat{k} \).

2. Find and classify the critical points of the function \( f(x, y) = 4xy - 2x^4 - y^2 \).

3. Find the first octant point on the surface \( xyz = 8 \) that is closest to \((0, 0, 0)\). (First octant = \( x, y, z \) all positive).

4. Find the equation of the tangent plane to the surface \( xy^2 + 2xyz - e^{xz} = 8 \) at the point \((1, 3, 0)\).

5. Use linear approximation to estimate \( \sqrt{(3.1)^2 + (4.2)^2 + (11.7)^2} \).

6. Find the highest point on the surface \( z = 4xy - x^4 - y^4 \).

7. Suppose that \( r = uvw - u^2 - v^2 - w^2 \), \( u = y + z \), \( v = x + z \), \( w = x + y \). Find \( \frac{\partial r}{\partial x} \).

8. Find \( \frac{\partial z}{\partial x} \) supposing that \( z = f(x, y) \) satisfies the equation \( xyz = \sin(x + y + z) \).

9. Show that the sphere \( x^2 + y^2 + z^2 = r^2 \) and the cone \( z^2 = a^2x^2 + b^2y^2 \) are orthogonal (that is, have perpendicular tangent planes) at every point of their intersection. (Fig. 12.8.12).

10. Find the maximum and minimum values that the function \( f(x, y, z) = 3x + 2y + z \) attains on the surface \( x^2 + y^2 + z^2 = 1 \).