Math 220 AD9 Spring 2009 Worksheet 7

1. What can the limit be if it "looks like" $\frac{0}{0}$?

$$\lim_{x\to 3} \frac{2x-18}{\sqrt{x-5}-2}, \qquad \lim_{x\to -1} \frac{(x+1)^2}{1+\frac{1}{x}}, \qquad \lim_{x\to 0} \frac{\sqrt{x^2}}{|x^5|}.$$

   What can the limit be if it "looks like" $\frac{\infty}{\infty}$? Find examples that illustrate the possibilities.

2. Sketch the graph of a function such that $\lim_{x\to\infty} f(x) = -1$ and $\lim_{x\to -\infty} f(x) = +1$.

3. Without using your calculator, analyze the left- and right-hand limits of the following functions at any point where they fail to exist. Use this information to draw (very rough) sketches of these functions.

$$f(x) = \frac{1}{x-1}, \qquad g(x) = \frac{x-1}{x+1}, \qquad h(x) = \frac{1-x^2}{x+2}, \qquad k(x) = \frac{|1-x|}{(x-1)^2}.$$

4. We will discover the behavior of $\frac{x^2-x+1}{2x^2+10x-7}$ as $x \to \infty$:
   Step 1: Figure out what the highest power of x is in the numerator and denominator.
   Step 2: Divide both the numerator and denominator by x to that power (in this case x^2).
   Step 3: Now take the limit of the numerator and denominator as $x \to \infty$. The number you get is the limit.

5. We will discover the behavior of $\frac{x^2-x+1}{2x^3+10x-7}$ as $x \to \infty$:
   Step 1: Figure out what the highest power of x is in the numerator and denominator.
   Step 2: Divide both the numerator and denominator by x to that power (in this case x^3).
   Step 3: Now take the limit of the numerator and denominator as $x \to \infty$. The number you get is the limit. What do we say if the numerator goes to zero and the denominator goes to infinity?

6. We will discover the behavior of $\frac{x^3-x^2+1}{2x^2+10x-7}$ as $x \to \infty$:
   Step 1: Notice that the highest power of x in the numerator is one more than that of the denominator.
   Step 2: Divide both the numerator and denominator by x to the power in the denominator (in this case x^2).
   Step 3: Now take the limit of the numerator and denominator as $x \to \infty$. In the numerator you should have an expression that looks like $x - 1$, and on the bottom you should have 2. In this case we say that the function has a slant asymptote at $y = 1/2x - 1/2$. Explain what this term means.

7. Now determine the behavior of the following polynomials as $x \to \infty$

(a) $f(x) = \frac{2x^3-5x^2+4x}{x^2-2x-7}$ (d) $f(x) = \frac{x^5+x^3-x^2+1}{2x^3+11x+1}$
(b) $f(x) = \frac{x}{x^2+1}$ (e) $f(x) = \frac{x^2+x}{2x+4}$
(c) $f(x) = \frac{x+1}{x-1}$ (f) $f(x) = \frac{x^4-x^3+x^2}{5x^4+10x-7}$

8. Evaluate the following limits:

$$\lim_{x\to\infty} \frac{2^x}{x^{100}}, \qquad \lim_{x\to-\infty} \frac{2^x}{x^{100}}, \qquad \lim_{x\to\infty} \frac{(0.9)^x x^{100}}{x^3+5}, \qquad \lim_{x\to\infty} \frac{\sqrt{x^4+5}}{x^3+x-10}, \qquad \lim_{x\to-\infty} \frac{x-5}{\sqrt{4x^2+1}}.$$

9. Evaluate the limits:

$$\lim_{x\to 0} e^{-\cot x}, \qquad \lim_{x\to 1} \arctan\left(\frac{x}{x^2-2x+1}\right), \qquad \lim_{x\to-\infty} e^{-x^2}, \qquad \lim_{x\to 0^+} e^{-2/x}.$$

10. When does a function have horizontal asymptotes and how do we find them?
When does a function have vertical asymptotes and how do we find them? Anything else to note here?
When does a function have slant asymptotes and how do we find them?

11. Q.54, p.108. What do we learn from this problem?

12. Determine the intervals on which the following functions are continuous.

$$f(x) = \log(4-x^2), \qquad g(x) = \frac{1}{1-x^2}, \qquad h(x) = \frac{1}{1+x^2}, \qquad k(x) = \frac{2-x}{x^2-4}.$$

13. True or False: $\lim_{x\to a} f(x) = L$ means that if x_1 is closer to a than x_2 is, then $f(x_1)$ will be closer to L than $f(x_2)$ is. Be prepared to justify your answer with an argument or counterexample.

14. Questions 75-80 on p. 120. Provide justifications if the answer is "TRUE" and counterexamples if the answer is "FALSE".

Preparation for next time:

Read Section 2.1. Number 5, p. 156. Briefly justify your answer.