Math 220 AD9 Spring 2009 Worksheet 7

1. Evaluate the limits:
$$\lim_{x\to 0} \frac{1}{x} \sin \frac{x}{3}, \qquad \lim_{\theta\to 0} \frac{\tan 5x}{\tan -2x}.$$

2. Suppose a state's income tax code states that tax liability is 12% on the first $20,000 of taxable earnings and 16% on the remainder. Write down a piece-wise defined function $T(x)$ that captures this situation. Check that $\lim_{x\to 0^+} T(x) = 0$ and $\lim_{x\to 20{,}000} T(x)$ exists. Why is it important for these limits to exist?

3. Are there constants a and b that make the function $f(x)$ continuous?
$$f(x) = \begin{cases} ax^2 & \text{if } x < 1 \\ 4 & \text{if } x = 1 \\ -x^3 + bx & \text{if } x > 1 \end{cases}$$
Find a and b, or explain why they don't exist.

4. There are three parts to the definition of continuity. What are they? For each condition, find an example of a function which breaks that condition.

5. Explain the Intermediate Value Theorem in your own words. Use the Intermediate Value Theorem to show $\sqrt[4]{5}$ exists. How could you use the IVT and your calculator to estimate $\sqrt[4]{5}$?

6. Let $g(x) = x^4 - x^3 + x^2 + x - 1$. What are $g(0)$ and $g(1)$? What does this tell you about zeroes of $g(x)$? Explain.

7. True or False: you were once exactly three feet tall. Explain.

8. True or False: at some time since your birth, your weight in pounds exactly equalled your height in inches. Explain.

9. At halftime in the basketball game, your team had 32 points. True or False: at some time your team had exactly 25 points.

10. Q.54, p.108. What do we learn from this problem?

11. Determine the intervals on which the following functions are continuous.
$$f(x) = \log(4 - x^2), \qquad g(x) = \frac{1}{1 - x^2}, \qquad h(x) = \frac{1}{1 + x^2}, \qquad k(x) = \frac{2 - x}{x^2 - 4}.$$

12. Without using your calculator, analyze the left- and right-hand limits of the following functions at any point where they fail to exist. Use this information to draw (very rough) sketches of these functions.
$$f(x) = \frac{1}{x - 1}, \qquad g(x) = \frac{x - 1}{x + 1}, \qquad h(x) = \frac{1 - x^2}{x + 2}, \qquad k(x) = \frac{|1 - x|}{(x - 1)^2}.$$

13. Sketch the graph of a function such that $\lim_{x\to\infty} f(x) = -1$ and $\lim_{x\to-\infty} f(x) = +1$.

14. We will discover the behavior of $\frac{x^2-x+1}{2x^2+10x-7}$ as $x \to \infty$:
Step 1: Figure out what the highest power of x is in the numerator and denominator.
Step 2: Divide both the numerator and denominator by x to that power (in this case x^2).
Step 3: Now take the limit of the numerator and denominator as $x \to \infty$. The number you get is the limit.

15. We will discover the behavior of $\frac{x^2-x+1}{2x^3+10x-7}$ as $x \to \infty$:
Step 1: Figure out what the highest power of x is in the numerator and denominator.
Step 2: Divide both the numerator and denominator by x to that power (in this case x^3).
Step 3: Now take the limit of the numerator and denominator as $x \to \infty$. The number you get is the limit. What do we say if the numerator goes to zero and the denominator goes to infinity?

16. We will discover the behavior of $\frac{x^3-x^2+1}{2x^2+10x-7}$ as $x \to \infty$:
Step 1: Notice that the highest power of x in the numerator is one more than that of the denominator.
Step 2: Divide both the numerator and denominator by x to the power in the denominator (in this case x^2).
Step 3: Now take the limit of the numerator and denominator as $x \to \infty$. In the numerator you should have an expression that looks like $x - 1$, and on the bottom you should have 2. In this case we say that the function has a slant asymptote at $y = 1/2x - 1/2$.

17. Now determine the behavior of the following polynomials as $x \to \infty$
    (a) $f(x) = \frac{2x^3-5x^2+4x}{x^2-2x-7}$
    (b) $f(x) = \frac{x}{x^2+1}$
    (c) $f(x) = \frac{x+1}{x-1}$
    (d) $f(x) = \frac{x^5+x^3-x^2+1}{2x^3+11x+1}$
    (e) $f(x) = \frac{x^2+x}{2x+4}$
    (f) $f(x) = \frac{x^4-x^3+x^2}{5x^4+10x-7}$

18. Evaluate the limits:
$$\lim_{x\to 0} e^{-\cot x}, \qquad \lim_{x\to 1} \arctan\left(\frac{x}{x^2-2x+1}\right), \qquad \lim_{x\to-\infty} e^{-x^2}.$$

19. True or False: $\lim_{x\to a} f(x) = L$ means that if x_1 is closer to a than x_2 is, then $f(x_1)$ will be closer to L than $f(x_2)$ is. Be prepared to justify your answer with an argument or counterexample.

Preparation for next time:

Read section 1.5. Do problem 27, p. 118.