Math 220 AD9 Spring 2009 Worksheet 5

1. Find functions $f(x)$ and $g(x)$ such that $\lim_{x\to 3} f(x)$ exists and $\lim_{x\to 3} g(x)$ exists and where

   (a) $\lim_{x\to 3} \frac{f(x)}{g(x)}$ exists and equals $\frac{\lim_{x\to 3} f(x)}{\lim_{x\to 3} g(x)}$.

   (b) $\lim_{x\to 3} \frac{f(x)}{g(x)}$ exists but does *not* equal $\frac{\lim_{x\to 3} f(x)}{\lim_{x\to 3} g(x)}$.

   (c) $\lim_{x\to 3} \frac{f(x)}{g(x)}$ does not exist.

2. Evaluate the following limits if they exist, or explain why they do not exist. Use algebraic methods or sketch a graph by hand.

   (a) $\lim_{x\to 0} \frac{|x|}{x}$.

   (b) $\lim_{x\to 0} \frac{|x|}{x^2}$.

3. Show, by example, that $\lim_{x\to a} [f(x) + g(x)]$ may exist even though neither $\lim_{x\to a} f(x)$ nor $\lim_{x\to a} g(x)$ exist.

4. Show, by example, that $\lim_{x\to a} [f(x)g(x)]$ may exist even though neither $\lim_{x\to a} f(x)$ nor $\lim_{x\to a} g(x)$ exist.

5. (a) Let f be a function which is defined everywhere. What is the relationship between the graphs of $f(x)$ and $f(1/x)$? Sketch an example.

   (b) If it exists, evaluate $\lim_{x\to\infty} \sin(1/x)$.

   (c) If it exists, evaluate $\lim_{x\to 0} \sin(1/x)$.

6. Suppose that you know $f(x) \le e^x$ for all x and also that $1 - x^4 \le f(x)$ for all x. What can you say about $\lim_{x\to 0} f(x)$? $\lim_{x\to 1} f(x)$?

7. Evaluate the following limits, if they exist. Justify in your most lawyerly fashion, your steps by naming the limit rules you're using. Don't forget to factor wherever you can and to try any algebraic tricks you can before deciding that a limit doesn't exist.

   (a) $\lim_{x\to 1} \left(x^2 + x + 1\right)$

   (b) $\lim_{x\to 1} \left((x^2 + 1)(x - 3)\right)$

   (c) $\lim_{x\to 3} \left(\sqrt{6 + x}\right)$

(d) $\lim_{x\to 2} \frac{x^2+2x-1}{x+3}$

(e) $\lim_{x\to 3} \frac{x^2-x-6}{x-3}$

(f) $\lim_{x\to 1} \frac{x^5-1}{x-1}$

(g) $\lim_{x\to -4} \frac{x^3+4x^2-9x-36}{x^2-16}$

(h) $\lim_{x\to 9} \frac{x-9}{\sqrt{x}-3}$

(i) $\lim_{x\to 2} \frac{\frac{1}{2}-\frac{1}{x}}{2-x}$

8. Given that $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$, try to find these other limits by performing any trigonometric tricks you can, or by judiciously changing variables.

(a) $\lim_{\theta\to 0} \frac{\theta}{\sin(-\theta)}$

(b) $\lim_{\theta\to 0} \frac{\sin^2\theta}{\theta}$

(c) $\lim_{\theta\to 0} \frac{1-\cos^2\theta}{\theta^2}$

(d) $\lim_{\theta\to 0} \frac{\sin 2\theta}{\theta}$

(e) $\lim_{\theta\to 0} \frac{1-\cos\theta}{\theta}$ (hint: look for something to multiply the top and bottom both by to get it more like c)

9. (a) Multiply out the following: $(\sqrt{t}-4)(\sqrt{t}+4)$

(b) Use the result of part (a) to simplify the following so there is no radical on the bottom:

$$\frac{16-t}{\sqrt{t}-4}$$

(c) Use the result of part (b) to solve the following:

$$\lim_{t\to 16} \frac{16-t}{\sqrt{t}-4}$$

10. (a) Write out the following with a common denominator: $\frac{1}{2} - \frac{1}{x}$

(b) Use the result of part (a) to rewrite the following:

$$\frac{\frac{1}{2} - \frac{1}{x}}{x - 2}$$

(c) Use the result of part (b) to solve the following:

$$\lim_{x \to 2} \frac{\frac{1}{2} - \frac{1}{x}}{x - 2}$$

Preparation for next time

Read Section 1.4. Do problem 11, p. 107.