Math 220 AD9 Spring 2009 Worksheet 40

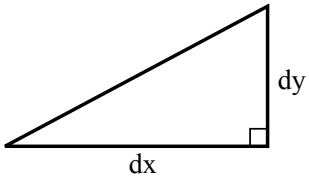
1. Use the method of cylindrical shells to find the volume of a sphere of radius r. What region are you rotating? Around which axis? Your answer should include pictures displaying a few typical cylindrical shells.

   Use mime or interpretative dance to explain the differences between solving this problem using disks and solving this problem using cylindrical shells. Your answer should have significant mathematical content.

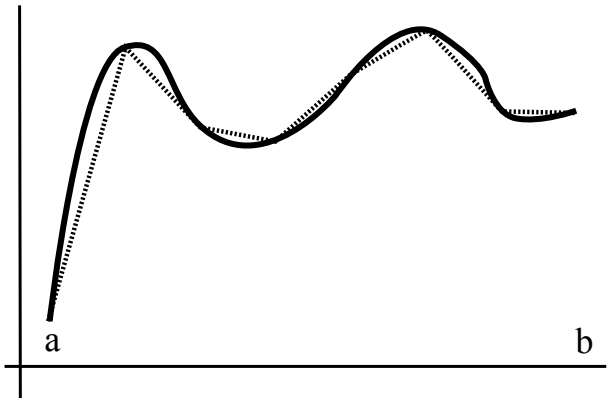
2. Use the method of cylindrical shells to find the volume of the region formed by rotating the region bounded by $y = x^2$, $y = x$, $0 \leq x \leq 1$ about the line $x = -1$. Include a sketch of some typical cylindrical shells.

3. Consider the following (right-angled) triangle:
   What is the length of the hypotenuse?

   Factor out a dx – it will be easier to use in this form.

   To approximate the length of the following curve, I could sum the lengths of the straight line segments pictured below. This naturally leads to a Riemann sum for the *arclength* of the curve. To get a more accurate estimate, I could increase the number of subintervals, which then leads to . . . ?

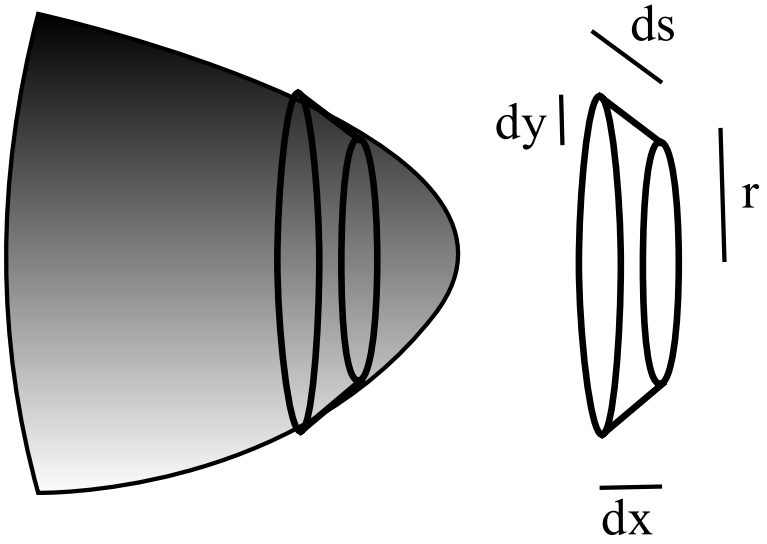
4. Compute the arclength of $y = 4x^{3/2} + 1$, $1 \leq x \leq 2$.

5. Compute the arclength of $y = e^{x/2} + e^{-x/2}$, $-1 \leq x \leq 1$.

6. Compute the arclength of $y = \frac{3}{4}\left(\frac{x^{4/3}}{2} - x^{2/3}\right)$ between $x = 1$ and $x = 8$.

7. What is the arclength of $y = \sqrt{4 - x^2}$ between $x = 0$ and $x = 2$? (Hint: No integration required.)

8. The following pictures show what happens when we rotate the graph of $x = 4 - y^2$ around the x-axis for $0 \leq x \leq 4$.



This time we are interested in the *surface area* of the shape on the left. To find this, we break the shape into pieces of the shape shown. From what we have already seen today, what is the "slant length" ds in terms of dx and dy? The surface area of the sloped part of this shape will be the circumference times the slant length. Use this to write down a definite integral for the surface area. Explain where your answer comes from.

9. Find the area of the surface generated by rotating the curve $y = x^3$, $0 \leq x \leq 2$, about the x-axis.

10. Use the above method to find the surface area of a sphere of radius r.

Preparation for next time

Read Section 5.5. There will be a preparation quiz.