Math 220 AD9 Spring 2009 Worksheet 40

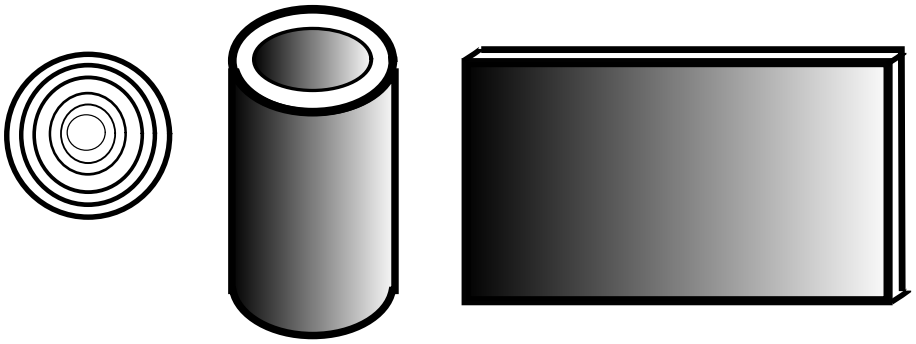
1. How do you decide what to do in a volume problem? You are given a region R, described by the intersection of various curves. You are told to rotate this around a (horizontal or vertical) line L. When you are finding the volume:

   - How do you decide whether to cut it into horizontal slices or vertical slices?
   - How do you decide whether to use "disks" or "washers"?
   - How do you find the radius?
   - What should your limits of integration be?
   - What should your definite integral be?

   Now apply your wisdom to the following:
   A region R is bounded by the curves $y = x^3$ and $y = x^2$. You obtain a solid by rotating this around the line $x = 3$. What is the volume of this solid? What if you had rotated it around the y-axis?

2. The Transamerica building in San Francisco is the 100th tallest building in the world, checking in at 853 feet. It's shape is that of a four sided pyramid. It's base is 180 ft by 180 ft. What is the volume of the building?

3. We can find the age of a (chopped down [choked sob]) tree by counting the numbers of rings in its trunk. We could also use the rings to find the volume of the tree trunk. The rings divide the tree trunk into hollow cylindrical "shells". We find the volume of each shell and then add up them to get the total volume.



   What is the volume of this cylindrical shell? If the shell is very, very, very, very thin (of thickness dx for example), then we could roll it out into a flat sheet as in the picture. If the height of the shell is h and the radius is r, and the thickness is dx, what is the volume of this cylindrical shell?

4. We break up the solid into cylindrical shells. To find the volume of each shell, we need to know the radius, the height, and the thickness (dx or dy). We then "add together" the shells by taking a definite integral.

   Consider the region bounded by $y = x^2$ and the $y = 1$, $-1 \leq x \leq 1$ revolved around the x axis. Sketch the region and a typical shell. What is the height? What is the radius of the shell? What is the volume?

   Notice how a (quick) sketch is an important part of solving these problems. What is the "formula" you should remember for these problems?

5. Use the method of cylindrical shells to calculate the volumes of the following solids of revolution. Let the regions be defined by the given equations.

   (a) $x = y^2$, $x = 4$, about the y-axis;

   (b) $y = x^3$, $y = 0$, $x = 2$, about the x-axis;

   (c) $y = e^{-x^2}$, $y = 0$, $x = 0$, $x = 1$, about the y-axis;

   (d) $y = x^2$, $y = x$, $0 \leq x \leq 1$ about the line $x = -1$.

   [Note that the third one cannot be done with the methods of disks or washers. Why not?]

6. Let's revisit the questions from the first problem on this worksheet. What are the corresponding questions and answers for the method of cylindrical shells?

7. Use the method of cylindrical shells to find the volume of a sphere of radius r. What region are you rotating? Around which axis?

Preparation for next time

Read Section 5.4. There will be a preparation quiz.