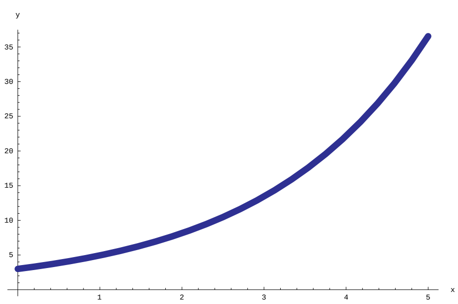
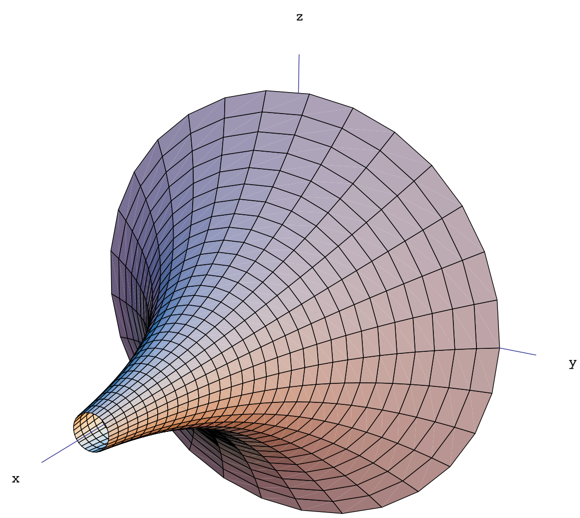
Math 220 AD9 Spring 2009 Worksheet 39

1. What is the area of the region bounded by the curves $x = 3y$ and $x = 2 + y^2$? Find two different ways of expressing this as an integral.

2. Consider the following graph of $y = 3e^{x/2}$ on the interval $[0, 5]$. We rotate it around the x-axis to get the tornado shaped solid on the right. What is the volume of this solid? How can we write the volume as an integral?





Riemann Sums to the rescue! (Thank you, Georg Friedrich Bernhard!)

- Break $[0, 5]$ into n subintervals. Draw the corresponding rectangle near $x = 3$ on the graph on the left.
- What does that give us in the picture on the right?
- What is the height of the rectangle on the left?
- What is the corresponding radius on the right?
- What is the volume of this slice on the right?
- How could we add up the volume of all these slices?
- Fill in the blanks: For every value of ____ between ____ and ____, we get a ________ of radius equal to ____ and which thus has ________ equal to ________. We can add up all of these ________ to get the ________ by ____.

3. We will show that the volume of a sphere of radius r is $4/3\pi r^3$.

   (a) Find the function whose graph is the top half of the circle with radius r centred at the origin.

(b) What shape is formed if we take that graph and spin it around the x-axis?

(c) To find area, we cut up a shape into rectangles. To find volumes, we slice the solid up into cross sections. What are the cross sections of this graph?

(d) Generate a function which gives the area of a cross section of the solid at the point $x = a$.

(e) Integrate that function (what are the appropriate bounds?) to calculate the volume.

(f) Does your answer make sense?

4. There does *not* exist one magic formula which answers all of these questions. You must use the *ideas* we have met today. In the question below, the region being revolved is the same in each case, but the shapes obtained and their volumes will be very different. If you do not understand the ideas, finding the right radii to use in your integral will be very difficult. As the last two problems illustrate, there may not be any radius involved at all. The key idea is adding up slices of area to get the volume.

5. Let R be the region bounded by $y = x^2$ and $y = 4$. Compute the volume of the solid formed by revolving R about the given line.

$$\begin{array}{llllll} (a) & y=6 & (b) & \text{the } y\text{-axis} & (c) & y=4 \\ (d) & y=-2 & (e) & x=2 & (f) & x=-4 \end{array}$$

6. The Transamerica building in San Francisco is the 100th tallest building in the world, checking in at 853 feet. It's shape is that of a four sided pyramid. It's base is 180 ft by 180 ft. What is the volume of the building?

7. Repeat the previous problem for the great pyramid of Giza which is 481 feet tall with a 756 ft square base.

Preparation for next time

Read Sections 5.3 and 5.4. Attend lecture. There will be a preparation quiz.