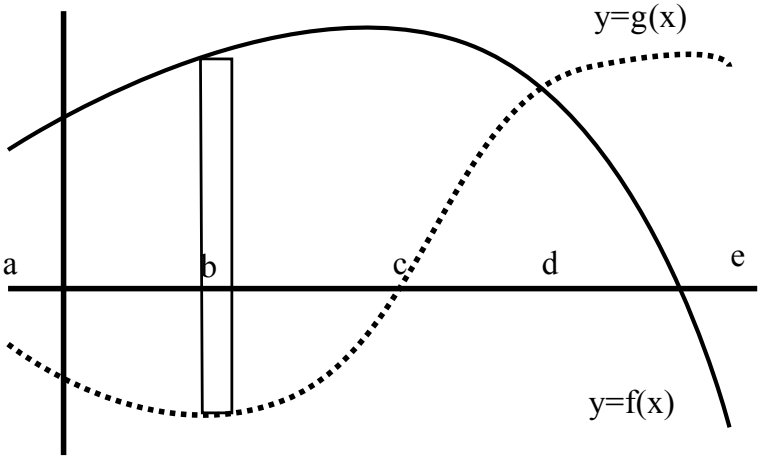
Math 220 AD9 Spring 2009 Worksheet 38

1. What is the area between the curves $y = f(x)$ and $y = g(x)$ on the interval $[a, c]$? To help, consider the following:

   - Divide the interval $[a, c]$ into n equal subintervals.
   - Draw rectangles (as in the graph) between the two curves and whose width is given by your n subintervals.
   - What is the height of the rectangle at $x = b$? What is the area of each rectangle?
   - What is the area between the two curves on the interval $[a, c]$?

   What is the area between the two curves on the interval $[a, e]$?

2. What is the area between the curves $y = x^3 - x$ and $y = 3x$ on the interval $[0, 4]$? What were the steps you went through to solve this problem? What was the reason for each step?

3. What is the area of the region bounded by the curves $y = 2x - x^2$ and $y = 2 - x^2$?

4. What is the area of the region bounded by the curves $y = x^3 - x$ and $y = 3x$? What were the steps you went through to solve this problem? What was the reason for each step?

5. Find the area of the region bounded by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$. Be careful!

6. Redo the last question in a way that involves doing only one integral.

   - Turn your page 90° counterclockwise and look at your sketch for the last problem. What advantages are there to this viewpoint?

- We usually think of $y = x - 1$ as describing y as a function of the variable x. However we could also rearrange it as $x = y + 1$. Now x is a function of the variable y.
- Say something similar about $y^2 = x + 6$.
- Write down an integral for the area where y is the only variable appearing.
- What are your limits of integration? How do you find them?
- Check that this agrees with your answer from above.
- In general, how do you decide whether to use x or y as your variable in the integral for the area?

7. Find the area of the region bounded by the curves $y = -1$, $y = x + 5$, $y = 2$ and $y^2 = x$.

8. Find the area of the region bounded by the curves $x = y^3 - y$, $x = 1 - y^4$.

9. If you wanted to solve the last question using integrals where x was the variable, what would you have to do?

10. Find the number b such that the line $y = b$ divides the region bounded by the curves $y = x^2$ and $y = 4$ into two regions with equal areas.

11. Find the area between the curves $y = \cos x$ and $y = \sin x$ on the interval $[0, 2\pi]$.

Preparation for next time

Read Sections 5.1 and 5.2. Attend lecture. There will be a preparation quiz.