Math 220, Spring 2009, Worksheet 36

1. What is the integral definition of $\ln x$?

   - What is the integral representation for $\ln a$?
   - What is the integral representation for $\ln b$?
   - Make the substitution $u = at$ in the integral for $\ln b$.
   - Add your answer to the integral for $\ln a$.
   - What have you shown?

2. Use Simpson's Rule (with $n = 8$) to estimate $\ln 9$.

3. Use the properties of logarithms to simplify the following expression:

   $$\ln\left(\sqrt[5]{\frac{x^3 \sin x}{\sqrt{x^2+9}}}\right)$$

   What is

   $$\frac{d}{dx}\left(\ln\left(\sqrt[5]{\frac{x^3 \sin x}{\sqrt{x^2+9}}}\right)\right)?$$

   What are the advantages of this approach?

4. Suppose $f(x) > 0$ for all x. What is

   $$\frac{d}{dx}\left(\ln f(x)\right)?$$

   What is the corresponding result for integrals?

5. Evaluate the following integrals:

   (a) $\displaystyle\int \frac{5x^4}{x^5+1}\,dx$

   (b) $\displaystyle\int_1^4 \frac{1}{\sqrt{x}(\sqrt{x}+1)}\,dx$

   (c) $\displaystyle\int \frac{\sec\theta\tan\theta}{1+\sec\theta}\,dx$

   (d) $\displaystyle\int_0^2 \frac{e^x}{e^x+1}\,dx$

   (e) $\displaystyle\int_1^3 \frac{x}{x+7}\,dx$

6. We have met a number of tips for choosing u when performing integration by substitution. What are they? Create examples that display how each of these rules can help you choose u.

7. Practice doing integrals by doing the following. Use u-substitution when appropriate.

   (a) $\displaystyle\int_0^1 (1-x)^9 \, dx$

   (b) $\displaystyle\int_0^1 \frac{e^{2x}}{e^x+1} \, dx$

   (c) $\displaystyle\int \frac{\cos \ln x}{x} \, dx$

   (d) $\displaystyle\int \frac{e^{\frac{2}{x}}}{x^2} \, dx$

   (e) $\displaystyle\int_0^1 \frac{e^x-1}{e^{2x}} \, dx$

   (f) $\displaystyle\int te^{t^2} \, dt$

   (g) $\displaystyle\int x\sqrt{2+3x} \, dx$

   (h) $\displaystyle\int \frac{x+2}{x^2+4x+3} \, dx$

   (i) $\displaystyle\int \frac{1}{x \log x} \, dx$

   (j) $\displaystyle\int \frac{(\log x)^2}{x} \, dx$

8. Use the midpoint and trapezoid rules with $n = 8$ to estimate $\ln 9$. How do these compare with your answer from Question 2? With the answer your calculator gives you?

Preparation for next time

There's an exam on Friday. Study!