# Math 220 AD9 Spring 2009 Worksheet 34

1. Now find the following definite integrals (don't forget to convert the limits of integration!):

   (a) $\displaystyle\int_0^1 \frac{(\arctan x)^2}{1+x^2}\,dx$

   (b) $\displaystyle\int_0^1 x^3\left(x^4+1\right)^3 dx$

   (c) $\displaystyle\int_0^{\pi/6} \sin 2x \cos^3 2x\; dx$

   (d) $\displaystyle\int_1^2 \frac{1+\ln x}{x} dx$

   (e) $\displaystyle\int_{\pi^2/4}^{\pi^2} \frac{\sin\sqrt{x}\cos\sqrt{x}}{\sqrt{x}} dx$

   (f) $\displaystyle\int_1^2 \frac{e^{-1/x}}{x^2}\; dx$

2. Practice doing integrals by doing the following. Use $u$-substitution explicitly

   (a) $\displaystyle\int te^{t^2}\; dt$

   (b) $\displaystyle\int x\sqrt{2+3x}\; dx$

   (c) $\displaystyle\int \frac{x+2}{x^2+4x+3}\; dx$

   (d) $\displaystyle\int \frac{1}{x\log x} dx$

   (e) $\displaystyle\int \frac{(\log x)^2}{x} dx$

3. What is the area of the trapezoid with corners $(a,0),(a,f(a)),(b,0)$, and $(b,f(b))$?

4. Draw 4 copies of a sketch of the curve $f(x)=x^3-4x+5$ on the interval [-4,4].
   (a) On one of these pictures, draw the rectangles you would use to estimate the definite integral of this curve on [-4,4] using a Riemann sum with right endpoints and $\Delta x=1$.
   (b) Draw the shapes you would use to do the trapezoidal approximation to $\displaystyle\int_a^b f(x)dx$ with $\Delta x=1$.
   (c) Draw the shapes you would use to do the midpoint approximation with the same $\Delta x$.
   (d) What is Simpson's approximation?

5. Compute (by hand) the midpoint, trapezoidal and Simpson's rule approximations (with $n=4$) of

$$\int_1^3 \frac{1}{x}\; dx.$$

6. If f(x) is concave down, then any trapezoidal approximation to $\int_a^b f(x)\,dx$ will be:
(a) an overestimate
(b) an underestimate
(c) exactly correct

7. If f(x) is a line, then the midpoint approximation to $\int_a^b f(x)\,dx$ will be:
(a) an overestimate
(b) an underestimate
(c) exactly correct

8. Theorems 7.1 and 7.2 allow us to say how accurate our estimates for the definite integral are. Why is it plausible for the error term to depend on the second derivative (or the fourth for Simpson's Rule)? (If you're stuck, sketch curves for which the second derivative is small and for which the second derivative is large and draw the trapezoids used by the Trapezoid rule.)

9. In what situations would numerical integration be used? Why not always use the Fundamental Theorem of Calculus?

10. (a) Find the approximations $T_{10}$(= the trapezoidal approximation with n=10) and $M_{10}$ (= the midpoint approximation with n=10) for the integral $\displaystyle\int_0^2 e^{-x^2}\,dx$.
(b) Estimate the errors in the approximation of part (a)
(c) How large do you have to choose $n$ so that the approximations $T_n$ and $M_n$ to the integral in part (a) are accurate to within 0.00001?

## Preparation for next time

For Wednesday, read section 4.8. There will be a preparation quiz for your Math 199 grade. Also, the practice midterm exam is Tuesday, 7-9pm in 143 Altgeld.