

Math 220 AD9 Spring 2009 Worksheet 34

1. The following calculation is incorrect. How do you know the answer is wrong? Where is the mistake?

$$\int_{-1}^1 \frac{1}{x^2} dx = \frac{-1}{x} \Big|_{-1}^1 = -1 - (-1) = 0.$$

2. What is the area of the region bounded by $y = x^2$, $x = 2$ and the x -axis?
3. What is the derivative of $f(x) = \sin(e^x)$? What is $\int \cos(e^x) e^x dx$?
4. What is the derivative of $f(x) = (x^3 + x^2 + 1)^{10}$? What is $\int (x^3 + x^2 + 1)^9 (3x^2 + 2x) dx$?
5. What is the derivative of $f(u(x))$? Write down the corresponding result for integrals. Rewriting integrals so that you can use this result is called integration by substitution. Note: There are *two* steps to integration by substitution. The first is to make the substitution and rewrite *everything* (including the limits and dx) in terms of u . The second is to compute the new (and easier) integral.
6. There are three things to remember when choosing what substitution to make:
- Is one part the derivative of another part of the function? (Or nearly so?)
 - Replace the most complicated part by something simpler.
 - Remember you are “undoing” the chain rule.

7. Suppose that we want to evaluate the interval $\int_1^5 x^2 \sin(x^3) dx$.

(a) We will use u -substitution. Find the appropriate u by finding a function in the integral that looks like the derivative of another. What is that u and what is its du ?

(b) Now convert the integral from x 's to u 's by filling in all the blank spots below:

$$\int_{x=1}^{x=5} x^2 \sin(x^3) dx \implies \int_{u=\underline{\quad}}^{u=\underline{\quad}} \underline{\quad} \underline{\quad} du$$

8. Now find the following definite integrals (don't forget to convert the limits of integration!):

$$\begin{array}{ll}
 \text{(a)} \int_0^4 \frac{dx}{\sqrt{2x+1}} & \text{(d)} \int_1^2 \frac{1+\ln x}{x} dx \\
 \text{(b)} \int_0^1 x^3 (x^4+1)^3 dx & \text{(e)} \int_{\pi^2/4}^{\pi^2} \frac{\sin \sqrt{x} \cos \sqrt{x}}{\sqrt{x}} dx \\
 \text{(c)} \int_0^{\pi/6} \sin 2x \cos^3 2x dx & \text{(f)} \int_1^2 \frac{e^{-1/x}}{x^2} dx
 \end{array}$$

9. Practice doing integrals by doing the following. Use u -substitution explicitly

$$\begin{array}{l}
 \text{(a)} \int (x+1)^6 dx \\
 \text{(b)} \int \sqrt{2x+1} dx \\
 \text{(c)} \int t e^{t^2} dt \\
 \text{(d)} \int x \sqrt{2+3x} dx \\
 \text{(e)} \int \frac{x+2}{x^2+4x+3} dx \\
 \text{(f)} \int \frac{1}{x \log x} dx \\
 \text{(g)} \int \frac{(\log x)^2}{x} dx
 \end{array}$$

10. If you are finding an indefinite integral: You started with a function of x , so your answer should be a function of x . Rewrite u in your answer in terms of x .

11. If you are finding a definite integral, you *must* change the limits of integration.

Your answer should be a number – you can get this in two ways. You can fill in for your new u -limits or you can rewrite u in your answer in terms of x and then use your x -limits.

Preparation for next time

For Friday, read section 4.7. There will be a preparation quiz for your Math 199 grade.

Also, the last midterm exam is one week from Friday!