

## Math 220 AD9 Spring 2009 Worksheet 32

1. Sketch the function  $f(x) = \sin x$  on the interval  $[0, \pi]$ . Divide this into 8 subintervals and use the midpoints as your evaluation points. Sketch the function and the approximating rectangles. Write down and evaluate the corresponding Riemann sum. What does this estimate? How can you improve upon this estimate?
2. Sketch the function  $g(t) = t^2 - t + 1$  on the interval  $[1, 3]$ . Divide this up into  $n$  subintervals. Possible choices for your evaluation points include the right endpoints, the left endpoints, and the midpoints of the subintervals. Which one of these would you expect to give you an overestimate? An underestimate? Be most accurate? As  $n$  increases, will the choice of evaluation points affect your final answer? Explain.

If you were dealing with the decreasing function  $g(t) = 5 - t^3$ , what are the answers to the above questions?

3. What is the definition and meaning of the *definite integral*

$$\int_a^b f(x) dx?$$

Write down definite integrals connected to the first two problems on this worksheet and explain how they are relevant.

4. Find  $\int_{-2}^2 x dx$  using Riemann sums. Explain what your answer means in terms of area.
5. Suppose that  $h(t)$  is an odd function. What can you say about  $\int_{-5}^5 h(t) dt$ ? What if  $h(t)$  is an even function?
6. Find  $\int_{-1}^2 x^2 - 1 dx$  using Riemann sums. Explain your answer in terms of area.
7. With the aid of suitable pictures, explain why

$$\int_a^b f(x) dx + \int_b^c f(x) dx = \int_a^c f(x) dx, \quad \int_a^b f(x) \pm g(x) dx = \int_a^b f(x) dx \pm \int_a^b g(x) dx.$$

Simplify the following expressions:

$$\int_{-2}^3 e^x dx + \int_3^5 e^x dx, \quad \int_2^7 \ln x dx - \int_4^7 \ln x dx,$$
$$\int_1^5 \sin^2 x dx + \int_1^5 \cos^2 x dx, \quad \int_{-1}^2 x^2 dx + \int_3^5 x^3 dx.$$

8. What is  $\int_{-2}^0 \sqrt{4-x^2} dx$ ? Hint: sketch the graph.

9. Without evaluating the definite integrals, say which one is larger:

$$\int_1^5 x dx \quad \text{or} \quad \int_1^5 x^2 dx?$$

Justify your answer.

10. What is the average depth (in feet) of the following swimming pool?

$x$	$0 - 6$	$6 - 12$	$12 - 24$	$24 - 36$	$36 - 72$
$y$	3	4	5	6	9

11. What is the average value of the function  $y = \sqrt{4-x^2}$  on the interval  $[-2, 0]$ ? Why is it reasonable to call this number the average value of the function?

12. Write down definite integrals equal to the following:

(a) The area above the  $x$ -axis and below  $y = 9 - x^2$ .

(b) The area bounded by the  $x$ -axis and the curve  $y = x^3 - x$ .

(c) The position of a bus after 4 hours, if  $s(0) = 0$  and its velocity is  $40t - 10t^3$

(d) The amount of beer sold by midnight if the bar opens at 6pm and beer is being poured at a rate of  $80t$  beers per hour at time  $t$ .

13. Find the region whose area is equal to the limit:

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{\pi}{4n} \tan i\pi 4n.$$

What is the corresponding definite integral?

## Preparation for next time

For Wednesday, read sections 4.4. There will be a preparation quiz for your Math 199 grade.