Math 220 AD9 Spring 2009 Worksheet 30

1. Write out in full and then evaluate the following:

$$\sum_{i=1}^{5} i^2, \qquad \sum_{j=3}^{7} i, \qquad \sum_{k=1}^{4} 3k-2, \qquad \sum_{m=1}^{5} m^2$$

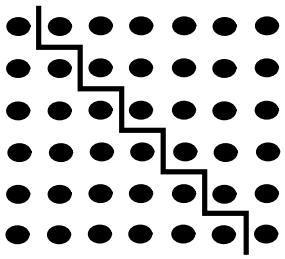
2. Write the following sums using sigma-notation.

$$a_4 + a_8 + a_{12} + a_{16} + a_{20} + a_{24}, \qquad 8 + 27 + 64 + 125 + 216 + 343$$

   How would you write the sum of the first fifty odd numbers using sigma-notation?

3. What is $\sum_{i=1}^{3} 4$? What is $\sum_{i=1}^{9} 3$? What is $\sum_{i=1}^{n} c$ for any constant c? Explain.

4. Count the dots in the following diagram in two ways:
   How many dots are in the rectangle?
   How many dots are in each "staircase"?
   How would you write this in sigma-notation?
   Suppose there were n rows in this diagram.
   What would the answers to the above questions be?
   Can you now find a formula for $\sum_{i=1}^{n} i$?



5. Similarly to the above (but in three dimensions!), we can show

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}.$$

   This is also proved in §4.2 of your textbook. Using this, what is the sum of the first eleven square numbers?

6. What is $\sum_{i=1}^{17} 4i^2 - 3i + 1$? (Hint: use the results from the last two problems.)

7. Evaluate the following limit

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\left(3\left(\frac{i}{n}\right)^2 + 4\left(\frac{i}{n}\right)\right)$$

8. Wiley E. Coyote is trying to drop an anvil off a cliff onto Roadrunner. He uses the following table showing the (downward) velocity of a falling Acme^{TM} anvil:

Time (s)	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5
Velocity (m/s)	10	14.9	19.8	24.7	29.6	34.5	39.4	44.3	49.2	54.1

   Using this information, how far does the anvil fall in 5 seconds?

9. Roadrunner escapes unharmed – Wiley's calculations were off and the anvil hit the desert floor before Roadrunner was near the cliff-face. Why?

   The actual velocity at each time t is given by $10 + 4.9t$ m/s. Wiley now repeats his calculations but correcting the velocity after each 0.1 seconds. Using sigma notation, express his new estimate. Evaluate this sum.

   What estimate would you get if you corrected the velocity every $\frac{1}{n}$ seconds? Express this using sigma-notation.

   How far does the anvil fall in 5 seconds?

10. Sketch the graph of the following function $y = 2x^2 + x + 3$ on the interval $[0, 1]$. What is the area under this curve between $x = 0$ and $x = 1$?

    We can evaluate this as follows.

    - Split the interval into 10 equal subintervals – what are the endpoints of these intervals?
    - Draw 10 rectangles whose bases are given by these 10 subintervals and whose tops just barely touch the graph of the curve.
    - Use sigma-notation to write down the sum of the areas of these rectangles.
    - Evaluate this sum.
    - Does this give you the area under the curve?
    - Repeat all the above steps only with n wherever there previously was a 10.
    - Take the limit as n tends to infinity – what does this give you?

11. What did we find in Question 7?

12. Do problem 26, p. 361 – what "meaning" does your final answer have?

13. How do you climb to the n^{th} rung of a ladder?

14. Using induction, show that

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}.$$

15. Use induction to show that $m^3 - m$ is divisible by three for $m \in \mathbf{N}$.

Preparation for next time

For Friday, read section 4.3. There will be a preparation quiz for your Math 199 grade. There will be a Math 220 quiz on sections 4.1 and 4.2.